Atomic and molecular transitions for neutrino and LDM detection

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in collaboration with Martin Bauer and Javier Perez-Soler

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QuMAP
Quantum Materials and
Astroparticle Physics

UNIVERSITY OF
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Contents

- Motivation: CνB and DM
- Generalised atomic transitions
- Pair absorption
- The future: molecules and ML

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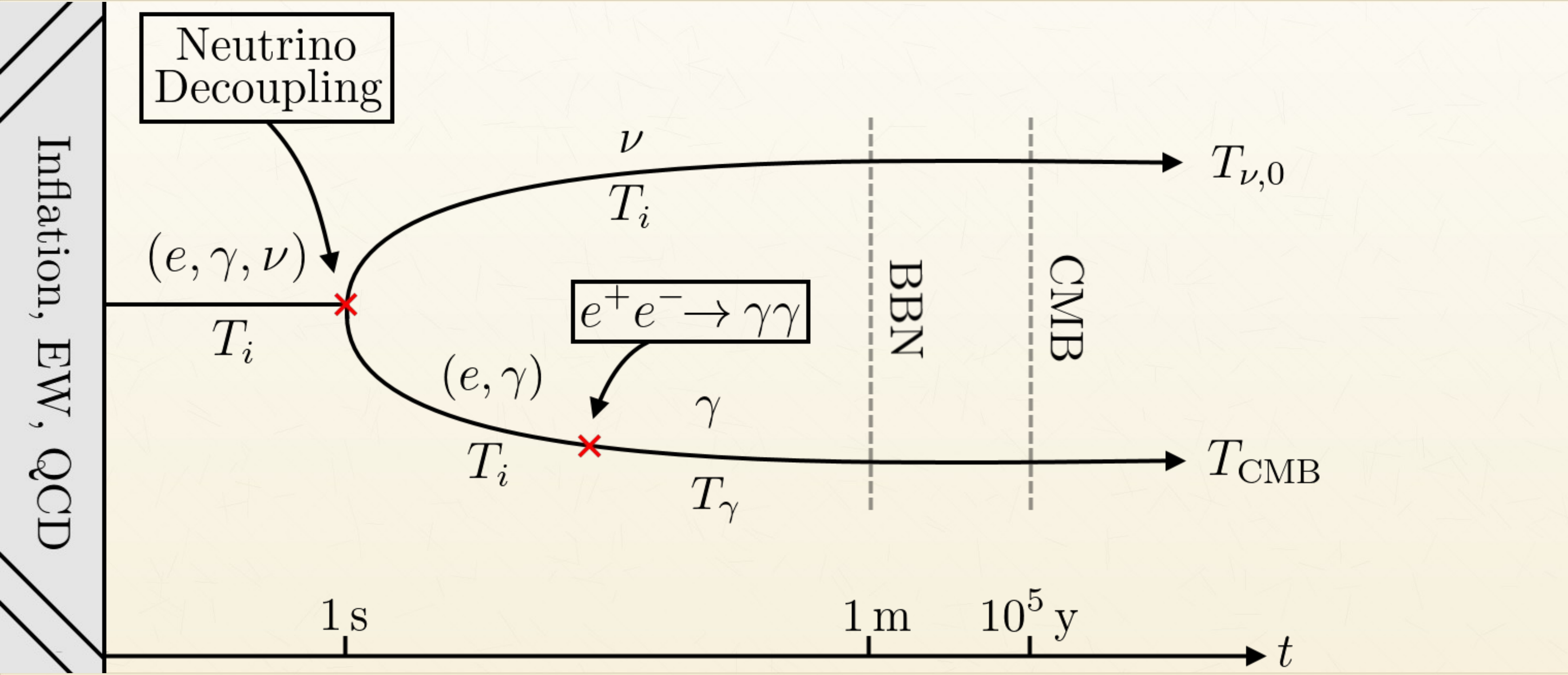
Motivation: The CνB



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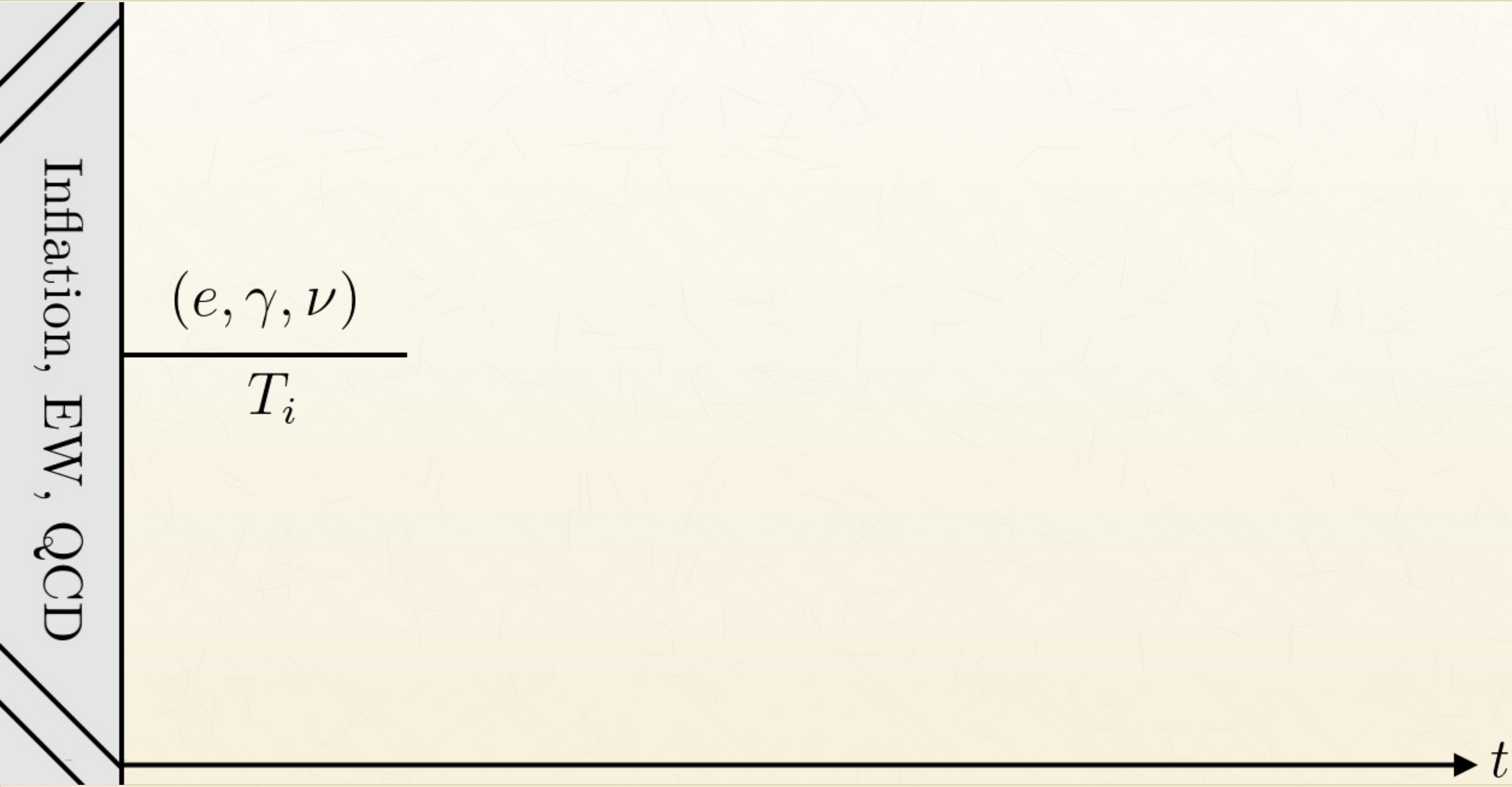

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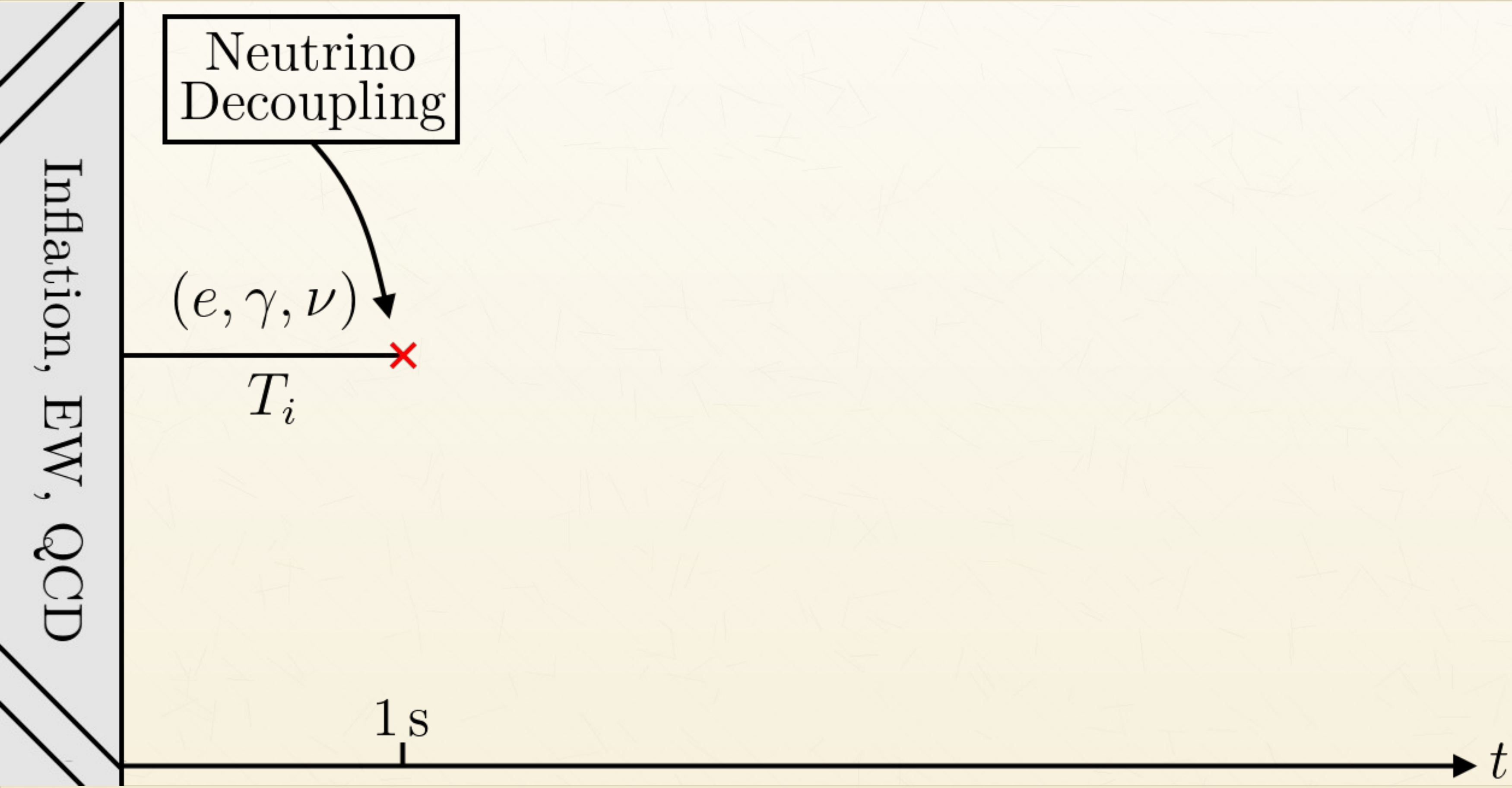

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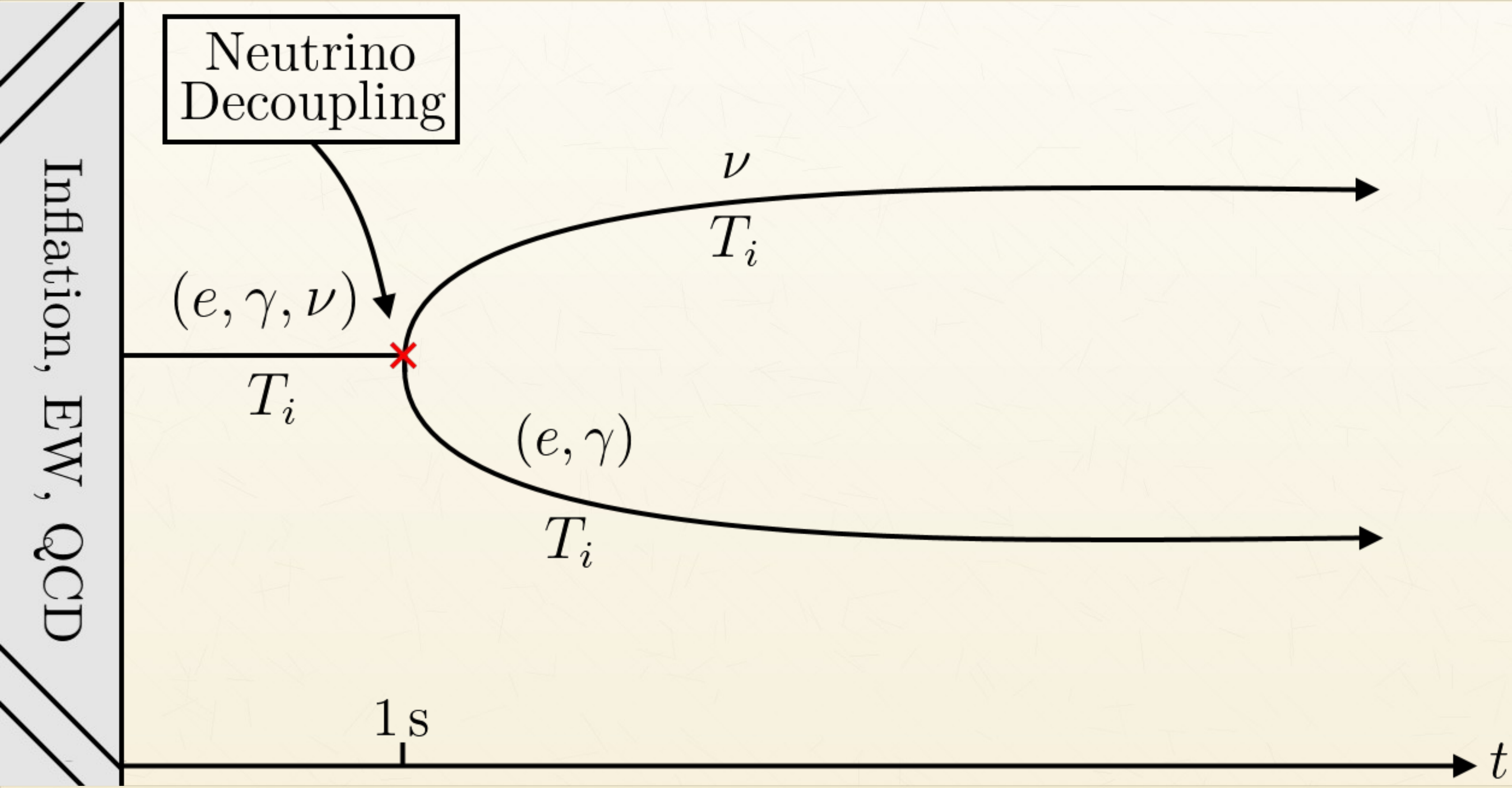

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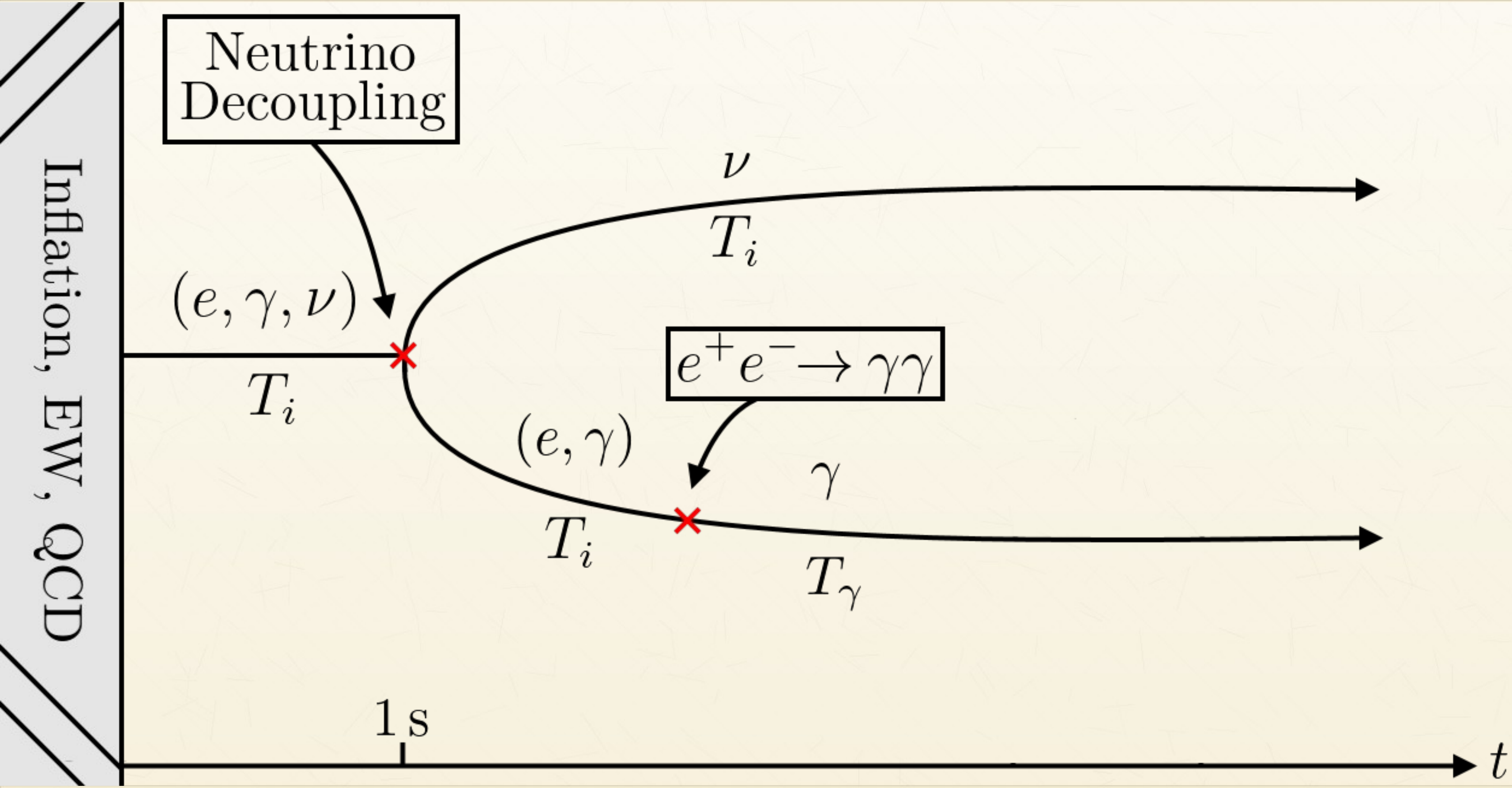

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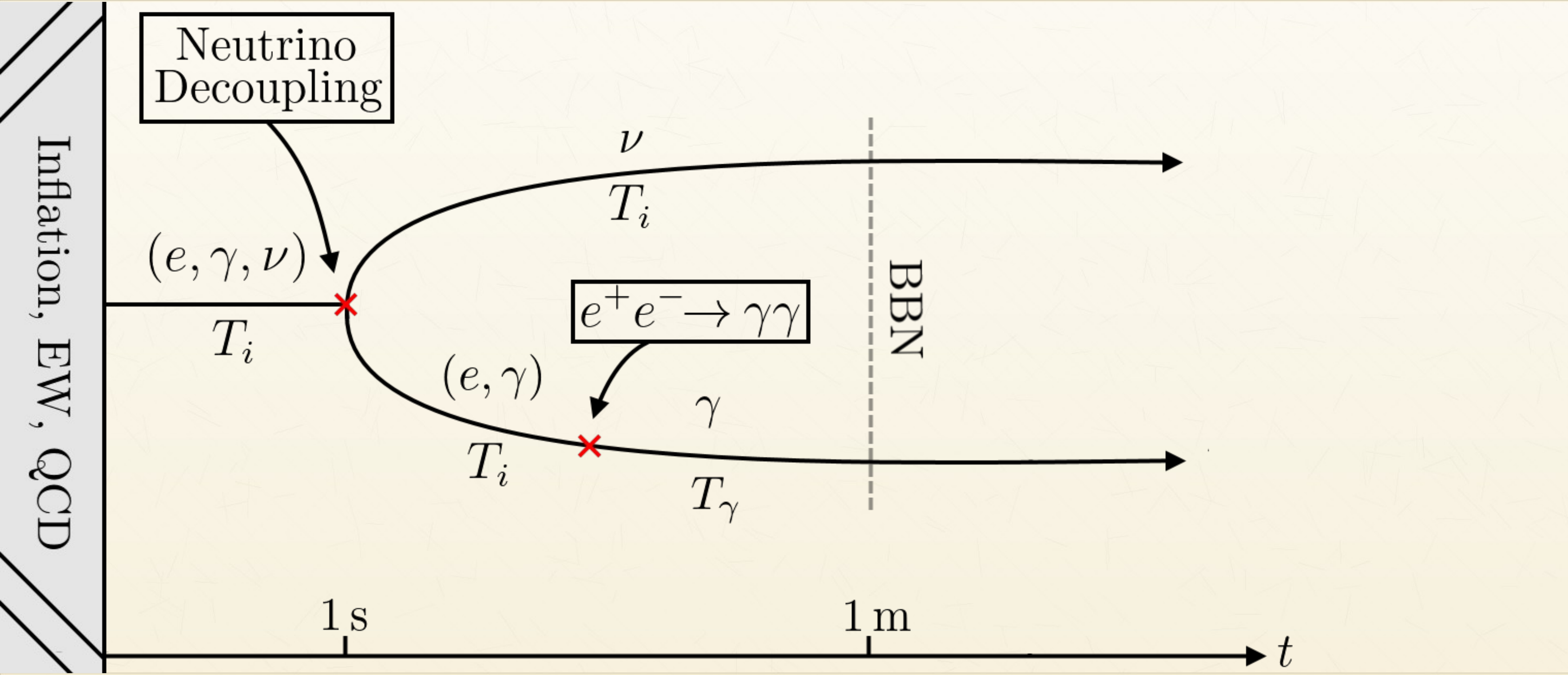

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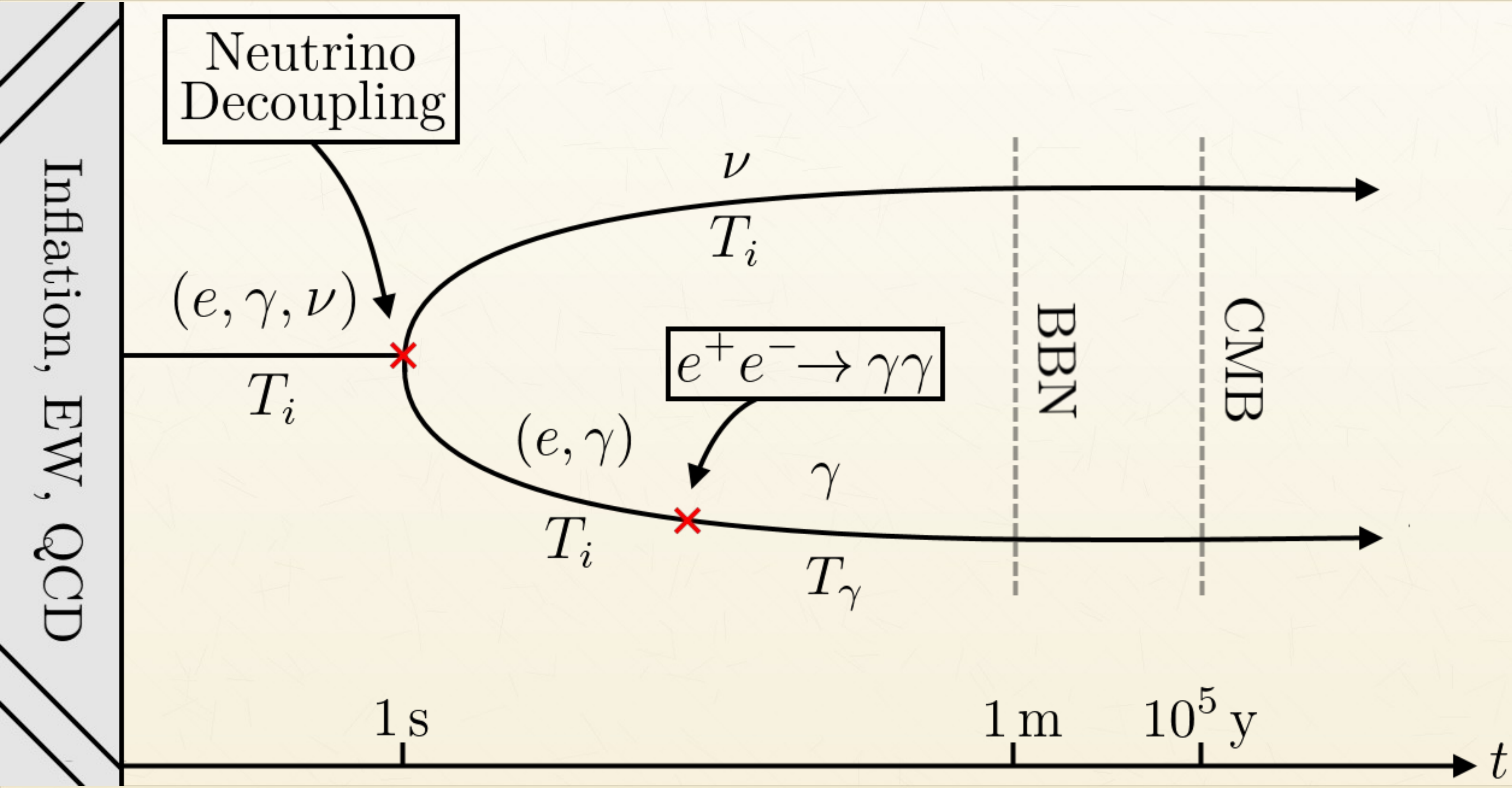

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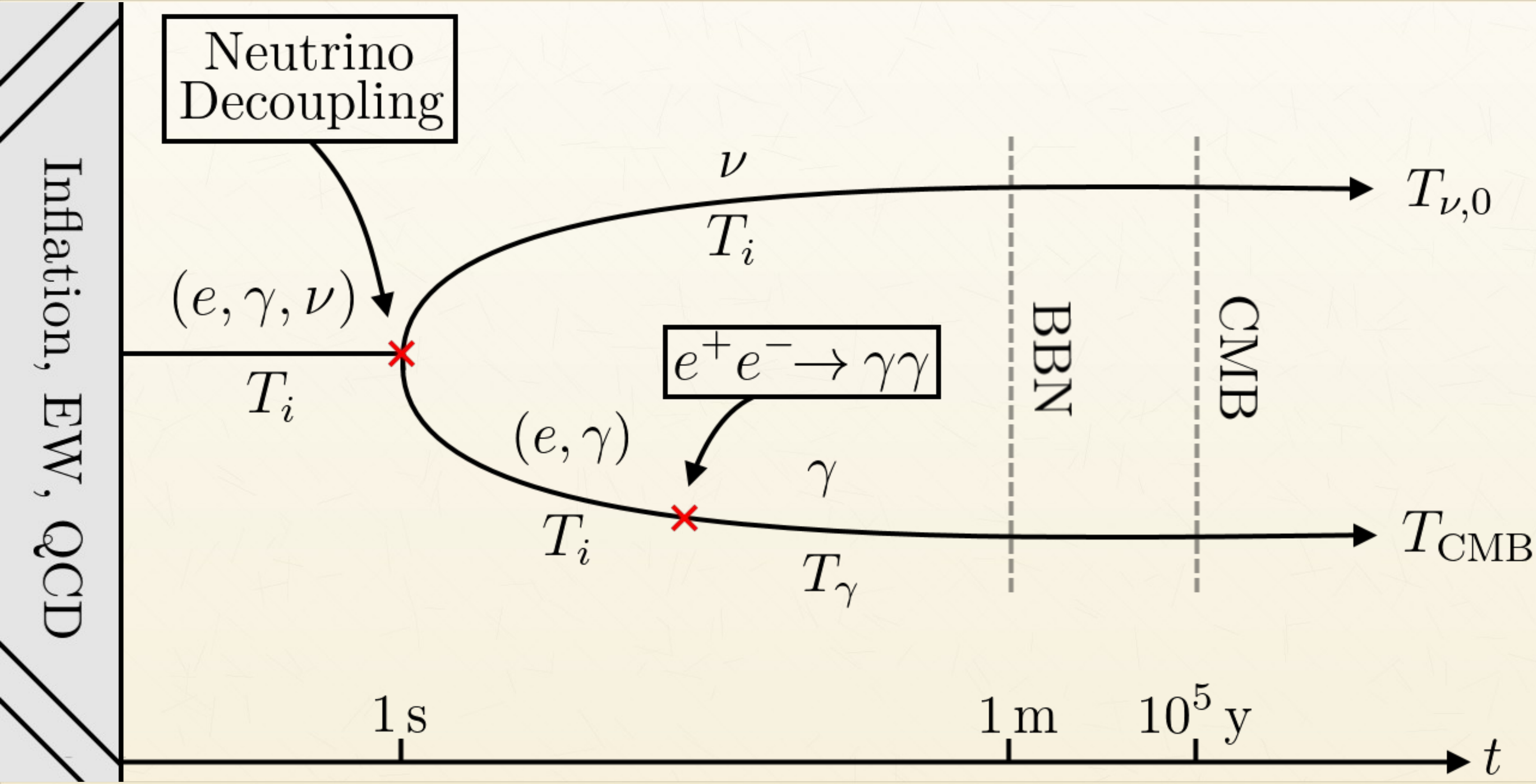

Motivation: The CνB

- Redshifted to temperature:

$$T_{\nu,0} = \left(\frac{4}{11}\right)^{\frac{1}{3}} T_{\mathrm{CMB}}$$

Motivation: The CνB

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$$T_{\nu,0} = 0.168\,\mathrm{meV}$$

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$$T_{\nu,0} = 0.168\,\mathrm{meV}$$

- Following a Fermi-Dirac distribution with:

$$n_\nu = 56\,\mathrm{cm}^{-3}$$

Motivation: The CνB

- Why do we care?

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Cosmology

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Neutrino physics

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- Verify ΛCDM

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- Dirac vs Majorana

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Neutrino physics

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- Absolute neutrino mass

Motivation: DM

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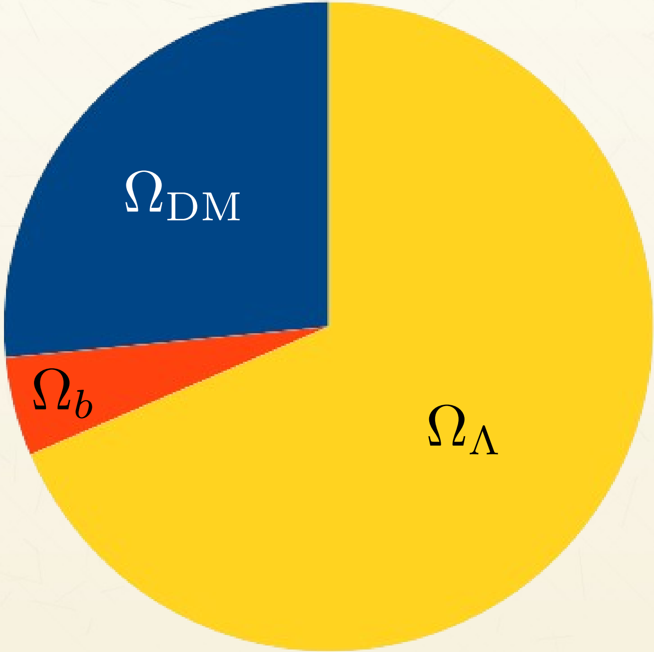

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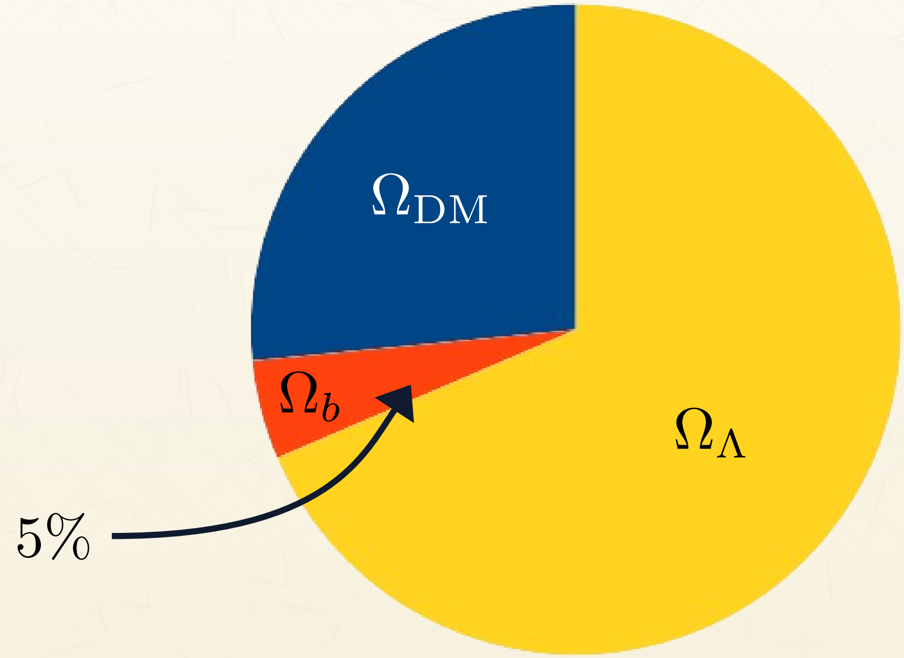

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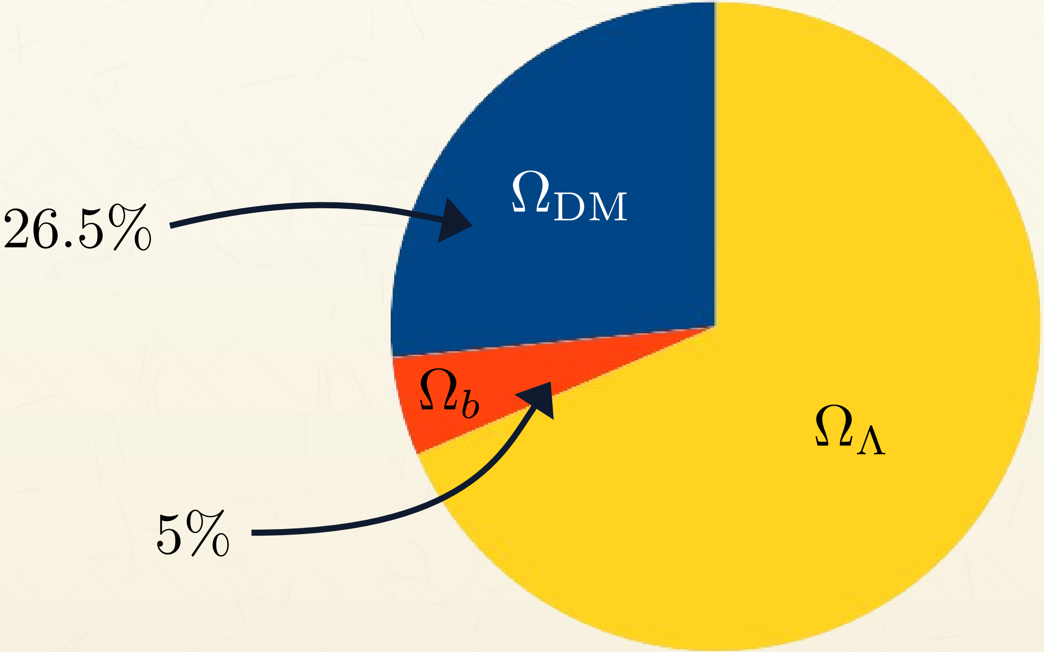

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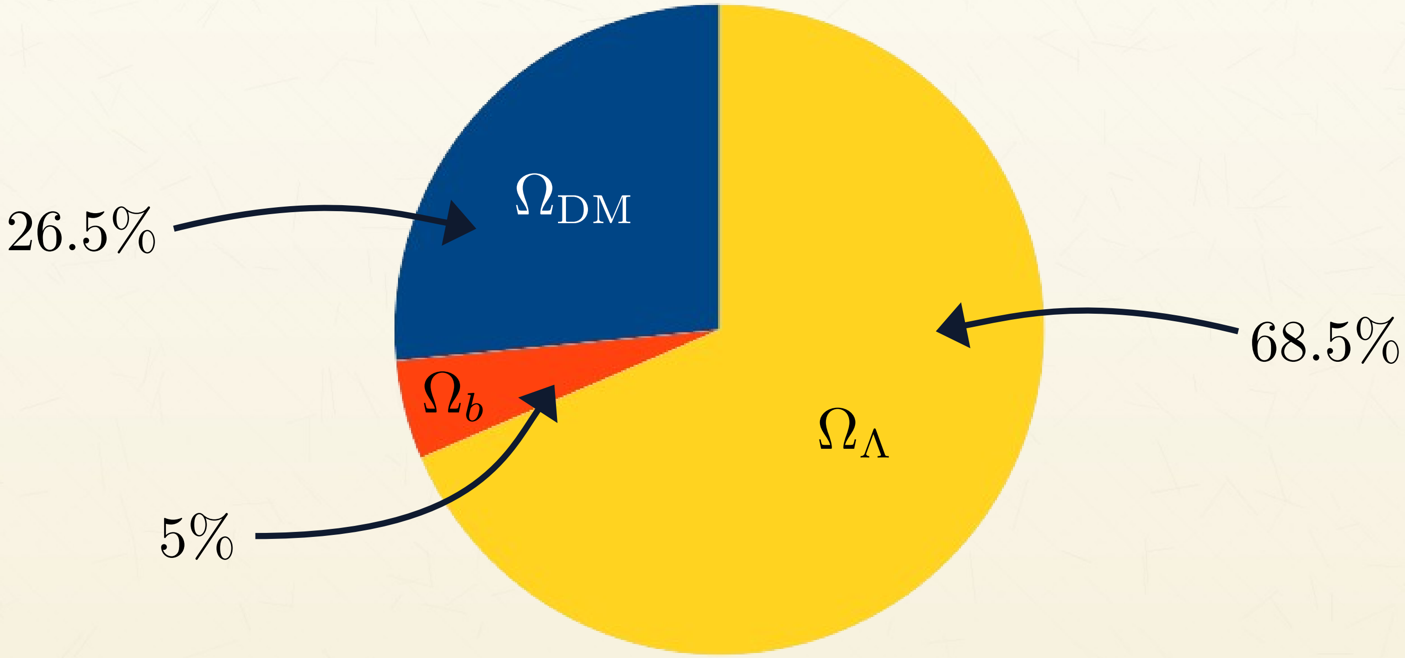

Motivation: Detection

- How do we detect light, cold relics?

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Elastic

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- Thresholdless

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$$R = \frac{n}{\Lambda^a} E_k^b$$

Inelastic

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Density

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UV scale

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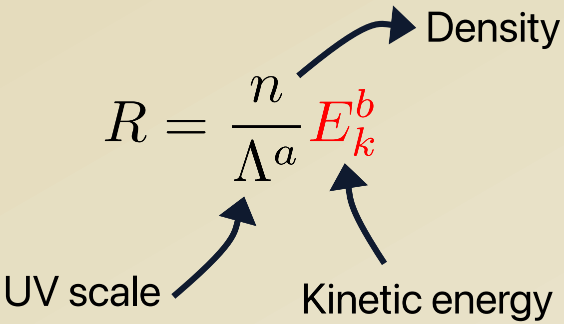

$$R = \frac{n}{\Lambda^a} E_k^b$$

Density

UV scale

Kinetic energy

Inelastic

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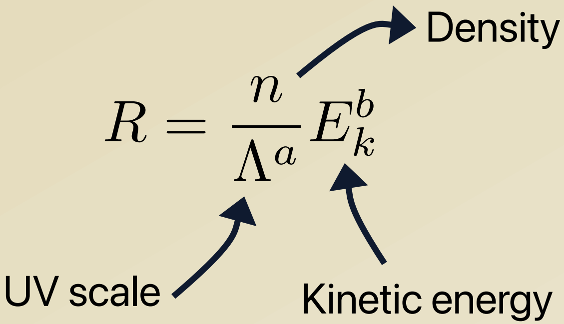

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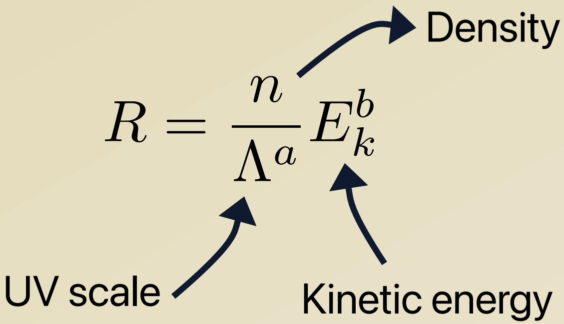

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Density

UV scale

Kinetic energy

Inelastic

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$$R = \frac{n}{\Lambda^a} E_{\text{thresh}}^b$$

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UV scale

Kinetic energy

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$$R = \frac{n}{\Lambda^a} E_{\text{thresh}}^b$$

Threshold energy

Motivation: Detection

- How do we detect light, cold relics?

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$$R_\nu = G_F^2 n_\nu E_{k,\nu}^2$$

Inelastic

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$$R = \frac{n}{\Lambda^a} E_{\text{thresh}}^b$$

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Fermi constant

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$$R_\nu = G_F^2 n_\nu m_\nu^2$$

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$$R_\nu = 3 \cdot 10^{-22}\,\mathrm{y}^{-1}$$

at $m_\nu = 10\,\mathrm{meV}$

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$$R_\nu = 3 \cdot 10^{-22}\,\mathrm{y}^{-1}$$

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Inelastic

- Threshold

$$R_\nu = 2 \cdot 10^{-16}\,\mathrm{y}^{-1}$$

at $m_\nu = 10\,\mathrm{meV}$

Motivation: Detection

- How do we detect light, cold relics?

Elastic

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$$R_\varphi = 3 \cdot 10^{-20}\,\mathrm{y}^{-1}$$

at $m_\varphi = 10\,\mathrm{meV}$

Inelastic

- Threshold

$$R_\varphi = 1 \cdot 10^{-7}\,\mathrm{y}^{-1}$$

at $m_\varphi = 10\,\mathrm{meV}$

Motivation: Detection

Focus of today's talk

- How do we detect light, cold relics?

Elastic

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$$R_\varphi = 3 \cdot 10^{-20}\,\mathrm{y}^{-1}$$

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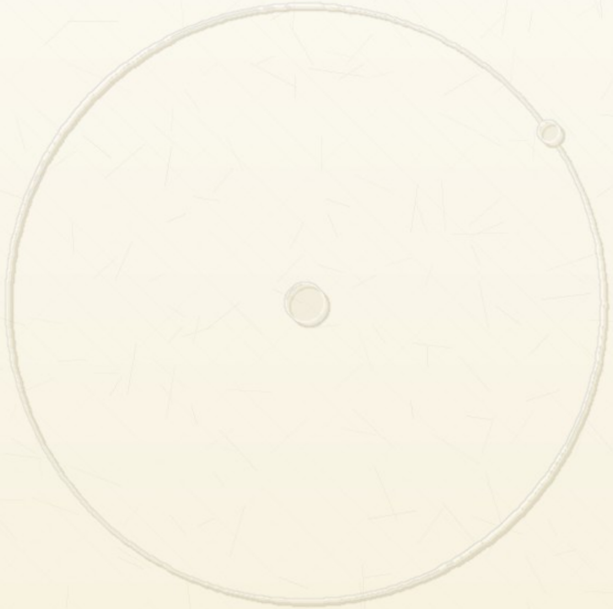
Generalised atomic transitions

- We need a system with:



Generalised atomic transitions

- We need a system with:
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Generalised atomic transitions

- We need a system with:
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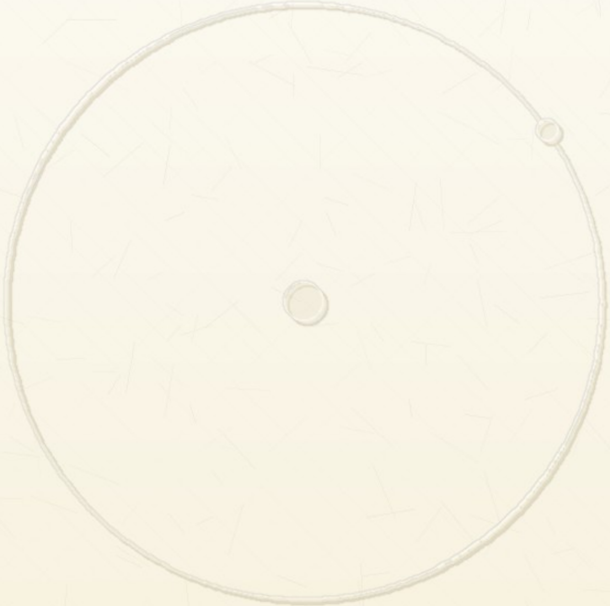
    - Clean signal



Generalised atomic transitions

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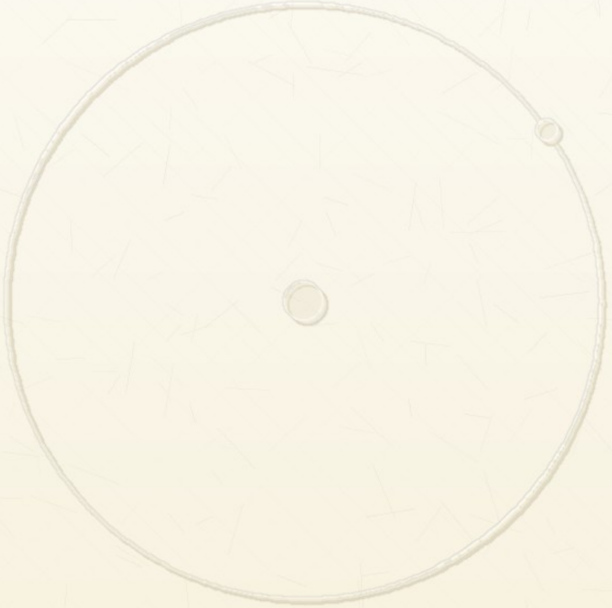
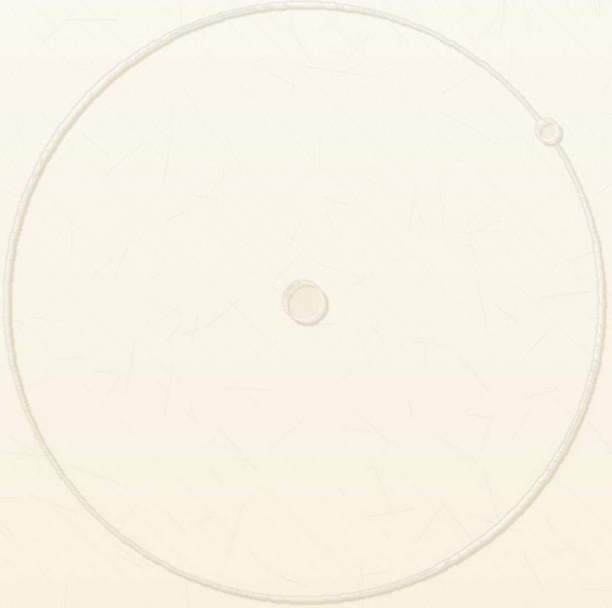
- Atomic systems!



Generalised atomic transitions

- We need a system with:
  - Tiny threshold, $\mathcal{O}(\mathrm{eV})$
  - Clean signal
- Atomic systems!
- But what are we looking for?



Generalised atomic transitions

- First option, new long range forces:

Generalised atomic transitions

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$$U(r) = \frac{Z\alpha_{\mathrm{EM}}}{r}$$

Generalised atomic transitions

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$$U(r) = \frac{Z\alpha_{\mathrm{EM}}}{r} \longrightarrow U(r) = \frac{Z\alpha_{\mathrm{EM}}}{r} + \beta\mathcal{R}(r)$$

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- Where, in general:

Generalised atomic transitions

- First option, new long range forces:

$$U(r) = \frac{Z\alpha_{\text{EM}}}{r} \longrightarrow U(r) = \frac{Z\alpha_{\text{EM}}}{r} + \beta\mathcal{R}(r)$$

- Where, in general:

$$\mathcal{R}(r) = \frac{1}{r^n} e^{-mr}$$

Generalised atomic transitions

- Second option, new transitions:

Generalised atomic transitions

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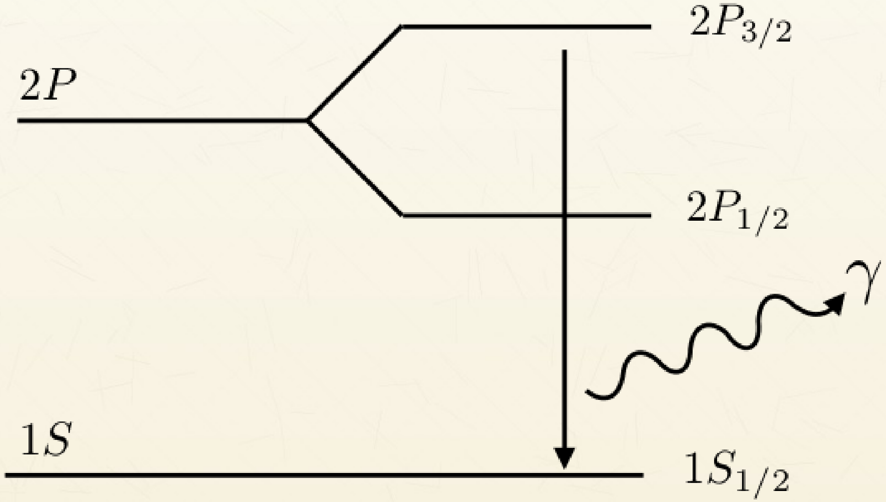
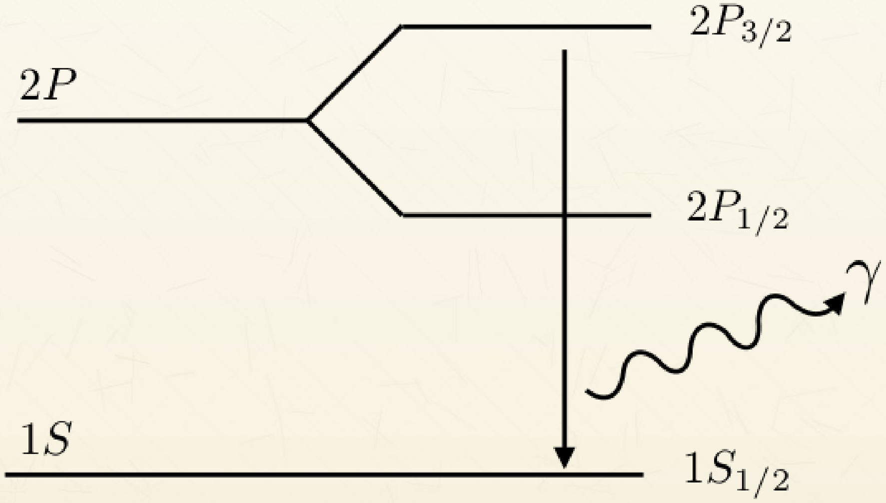
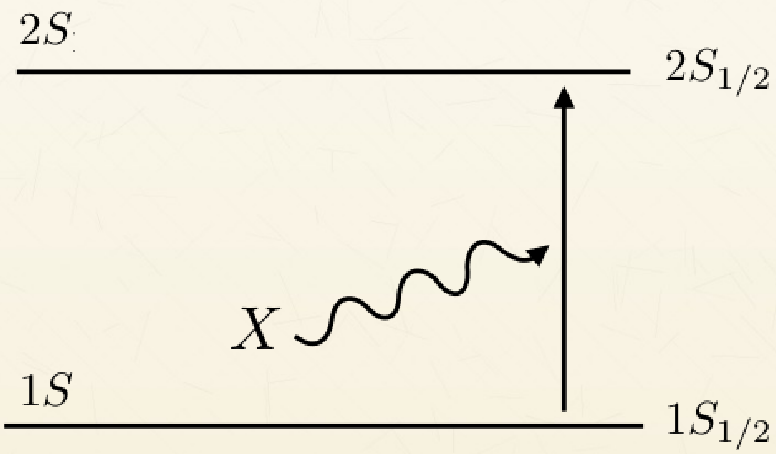


Generalised atomic transitions

- Second option, new transitions:





Generalised atomic transitions

- How do we compute these rates?

$$d\Gamma = 2\pi |\mathcal{M}_{fi}|^2 \delta(\dots) d\rho$$

Generalised atomic transitions

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Amplitude

Generalised atomic transitions

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Energy conservation

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Amplitude

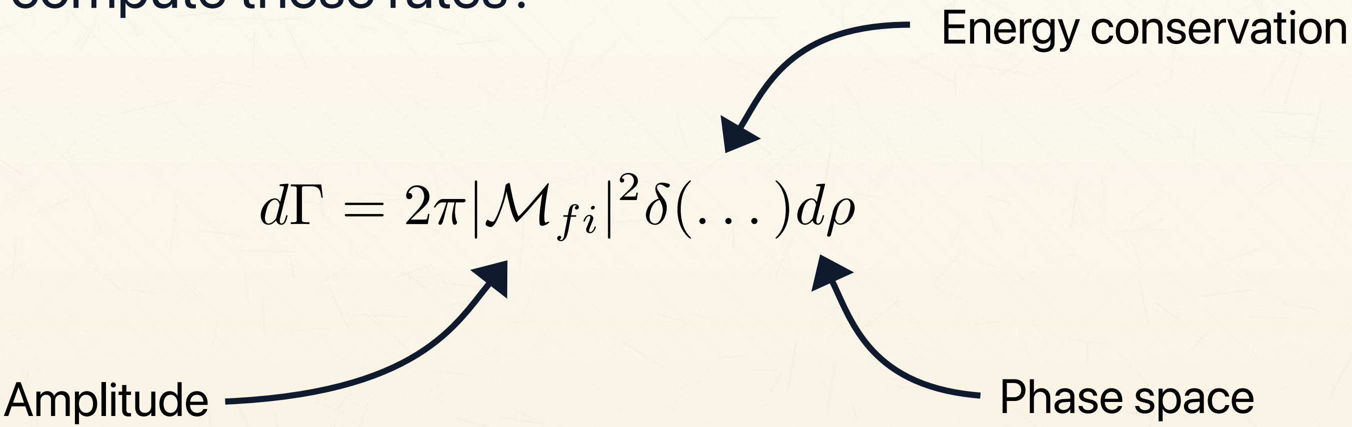
Generalised atomic transitions

- How do we compute these rates?

$$d\Gamma = 2\pi \underbrace{|\mathcal{M}_{fi}|^2}_{\text{Amplitude}} \overbrace{\delta(\dots)}^{\text{Energy conservation}} \underbrace{{\color{red} d\rho}}_{\text{Phase space}}$$

Generalised atomic transitions

- How do we compute these rates?



Energy conservation

$$d\Gamma = 2\pi|\mathcal{M}_{fi}|^2\delta(\dots)d\rho$$

Amplitude

Phase space

$$\mathcal{M}_{fi} = \langle f|H_{\text{int}}(\psi_e, A_\mu, \varphi, \dots)|i\rangle$$

Generalised atomic transitions

- We work with relativistic wavefunctions, but why?

Generalised atomic transitions

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- Because it’s easier…lets us use the language of QFT:

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- Because it's easier…lets us use the language of QFT:

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- More precise at large Z, and more transitions

Generalised atomic transitions

- What's the catch?

Generalised atomic transitions

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- Very computationally expensive:

Generalised atomic transitions

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    - Four component wavefunctions!

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Generalised atomic transitions

- What's the catch?

- Very computationally expensive:
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- Classical labelling (e.g. M1, E1, …) obscured

Generalised atomic transitions

- Let's count the terms: $N_{\text{terms}} = 1$

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$$\Gamma_L^{\{\mu\}} \in \{1, \gamma^5, \gamma^\mu, \gamma^\mu\gamma^5, \sigma^{\mu\nu}\}$$

Generalised atomic transitions

- Let’s count the terms: $N_{\text{terms}} = N_L \times N_\mu$

$$\mathcal{M}_{fi} = \langle f | \int d^3x \, \mathcal{H}_{\text{int}}(x) | i \rangle$$

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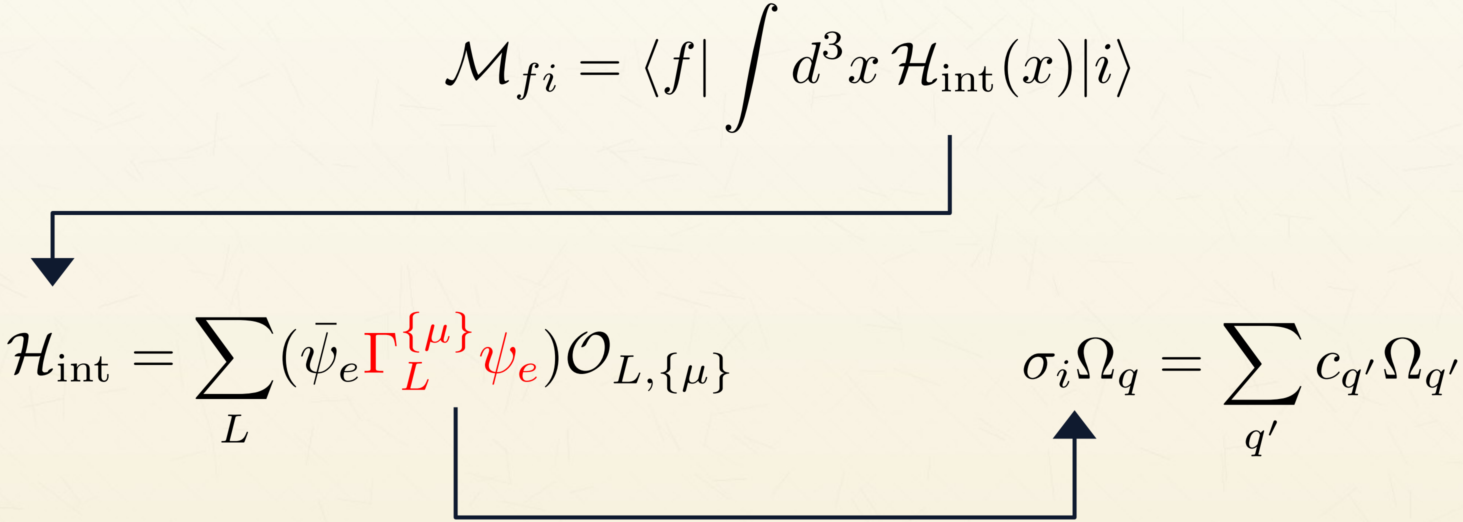

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Generalised atomic transitions

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Generalised atomic transitions

- Let’s count the terms: $N_{\text{terms}} = N_L \times N_\mu \times N_{q'} \times 2$

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- Example, QED:

Generalised atomic transitions

- Let's count the terms: $N_{\text{terms}} = 3 \times N_{\mu} \times N_{q'} \times 2$

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- Example, QED:

$$\mathcal{O}_{L,\{\mu\}} \in \{eA_{\mu}, \mu_B F_{\mu\nu}, \mu_E \widetilde{F}_{\mu\nu}\}$$

Generalised atomic transitions

- Let's count the terms: $N_{\text{terms}} = 3 \times 5 \times N_{q'} \times 2$

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Generalised atomic transitions

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Generalised atomic transitions

- Let's count the terms: $N_{\text{terms}} = \text{a lot}$

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Generalised atomic transitions

- Let's count the terms: $N_{\text{terms}} = \text{a lot}$

$$\mathcal{M}_{fi} = \langle f| \int d^3x\, \mathcal{H}_{\text{int}}(x) |i\rangle$$

- We really want the squared amplitude:

Generalised atomic transitions

- Let's count the terms: $N_{\text{terms}} = (\text{a lot})^2$

$$\mathcal{M}_{fi} = \langle f| \int d^3x \, \mathcal{H}_{\text{int}}(x) |i\rangle$$

- We really want the squared amplitude:

$$|\mathcal{M}_{fi}|^2 = |\langle f| \int d^3x \, \mathcal{H}_{\text{int}}(x) |i\rangle|^2$$

Generalised atomic transitions

- Let's count the terms: $N_{\text{terms}} = \sum (\text{a lot})^2$

$$\mathcal{M}_{fi} = \langle f| \int d^3x\, \mathcal{H}_{\text{int}}(x) |i\rangle$$

- And what if we want to sum over polarisations?

$$|\mathcal{M}_{fi}|^2 = \sum_{m,m'} |\langle f| \int d^3x\, \mathcal{H}_{\text{int}}(x) |i\rangle|^2$$

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Radial integral: sets the scale

Angular integral: selection rules

Generalised atomic transitions

CINCO

https://gitlab.com/JShergold/cinco

Generalised atomic transitions

$\Gamma_L^{\{\mu\}}$

CINCO

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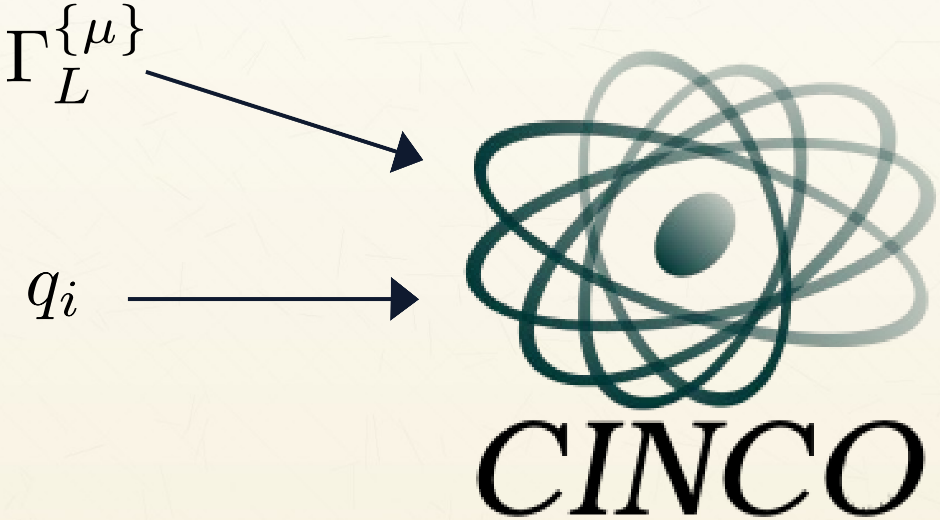
Generalised atomic transitions



$\Gamma_L^{\{\mu\}}$

q_i

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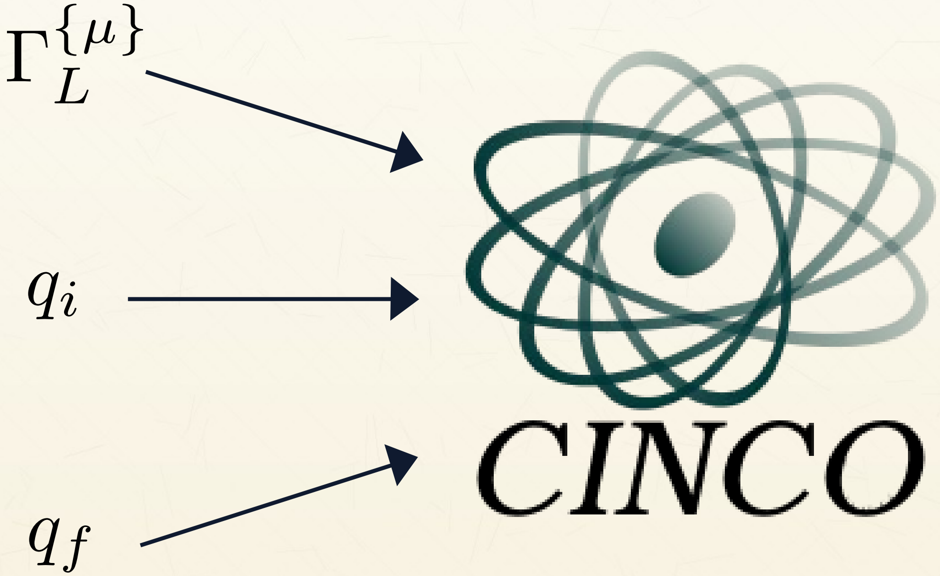

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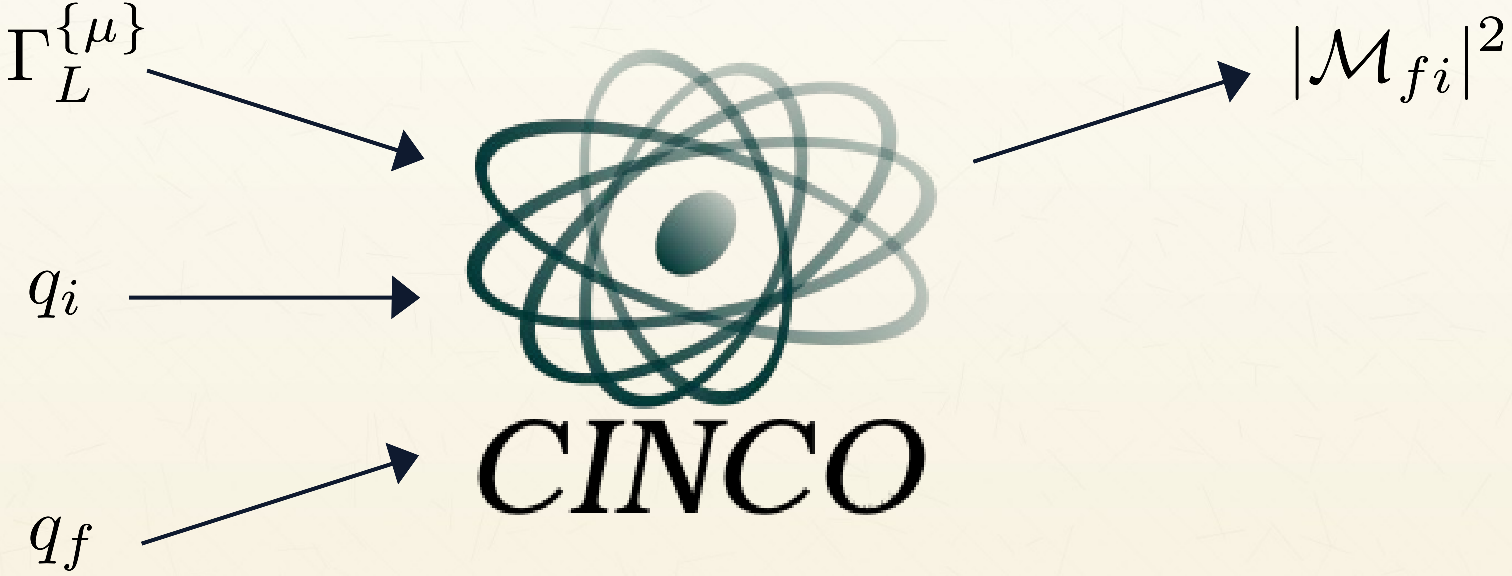

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CINCO

$|\mathcal{M}_{fi}|^2$

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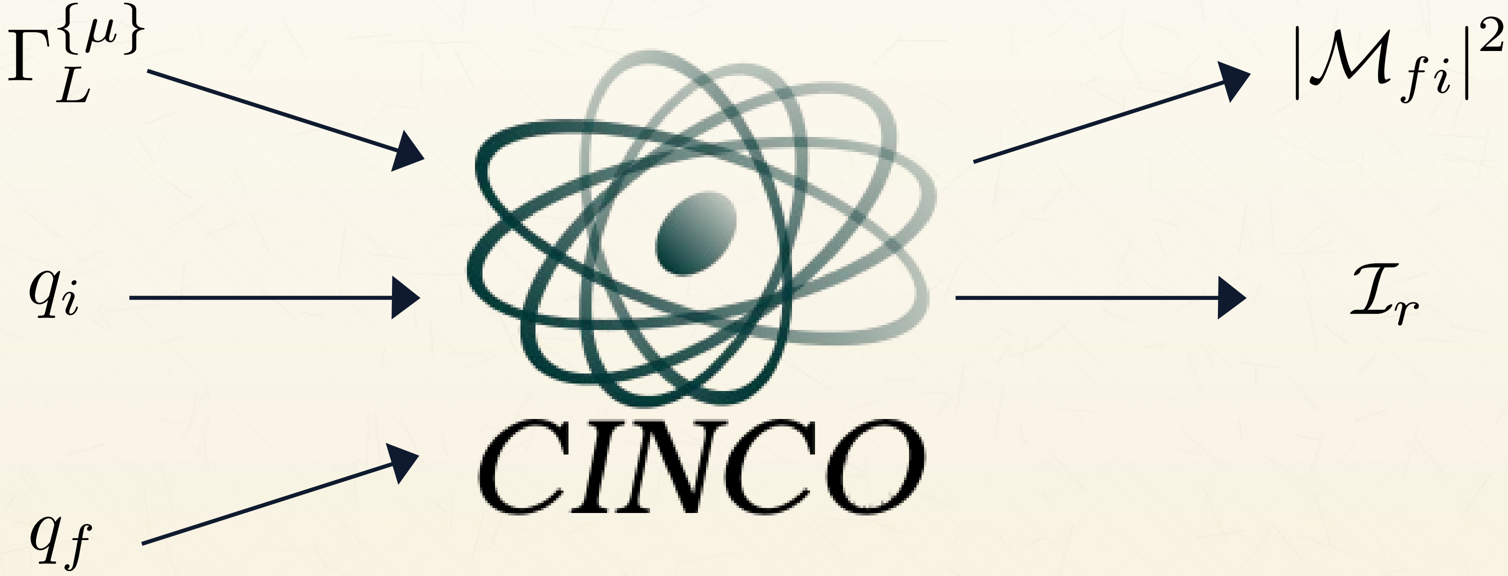

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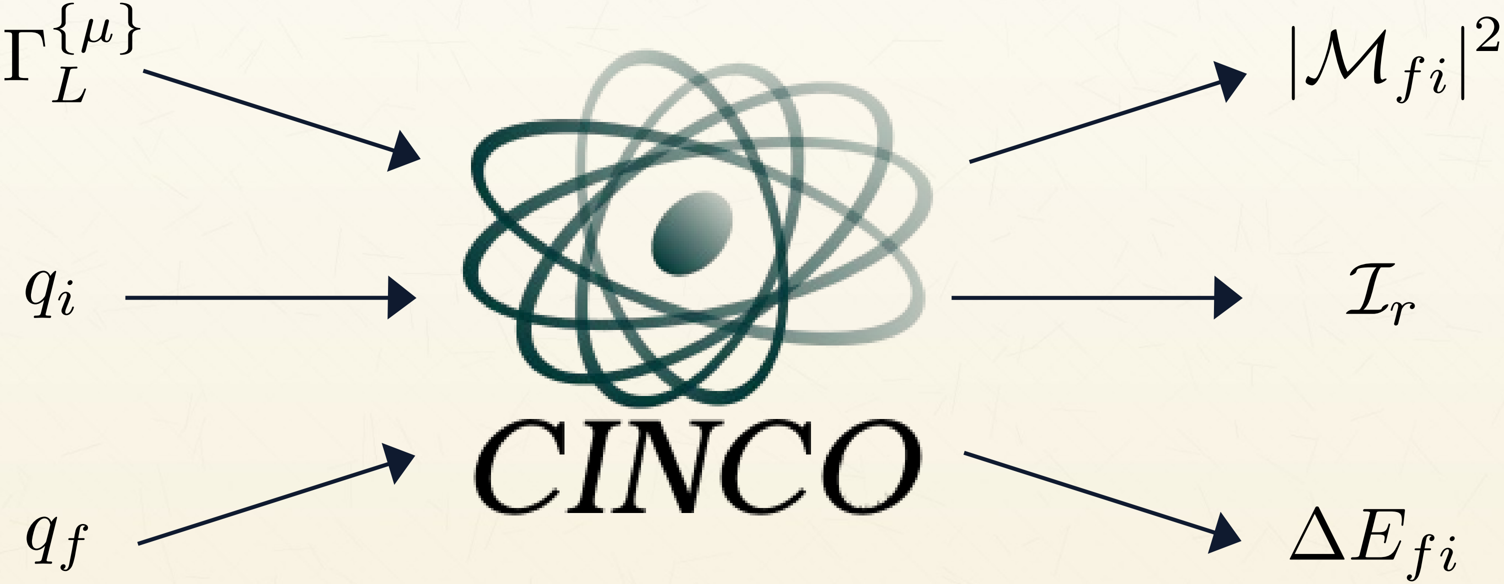

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ΔE_{fi}

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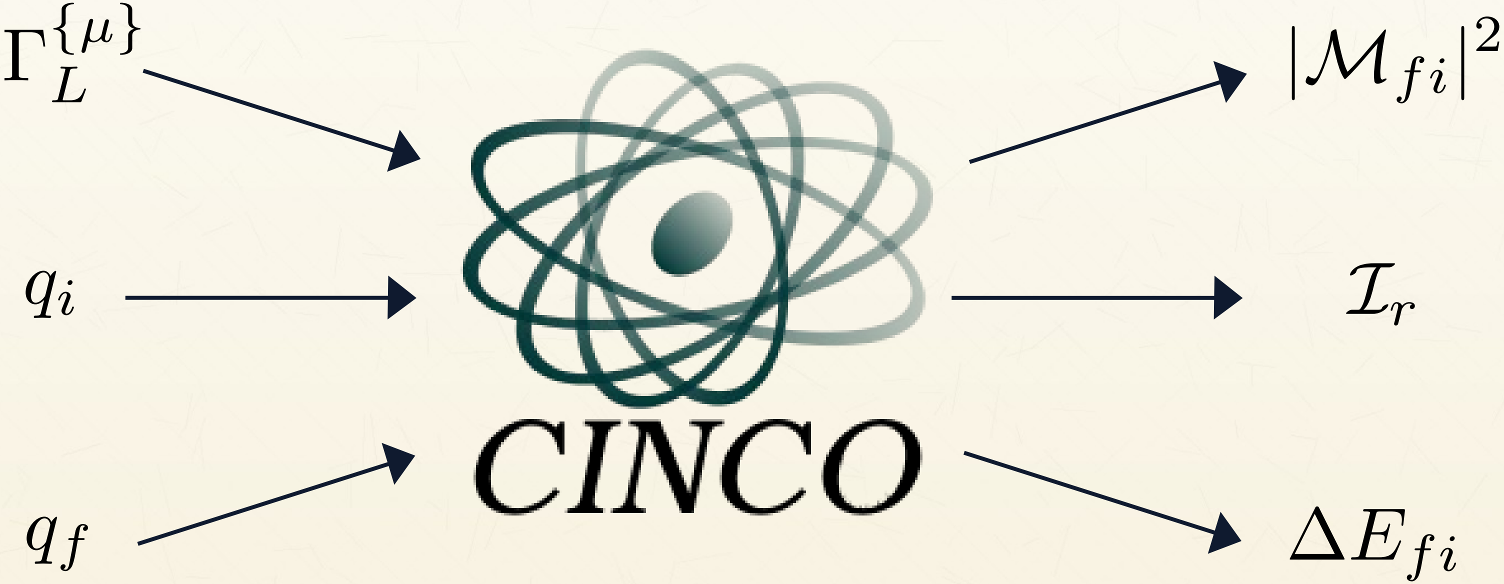

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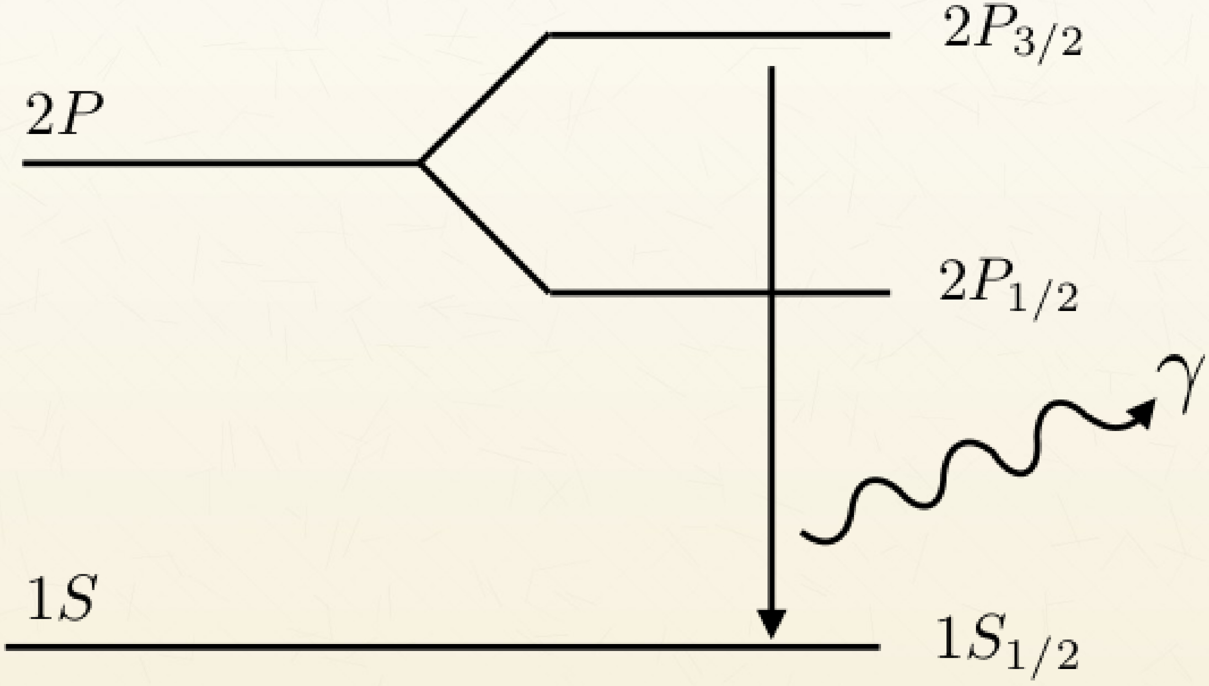

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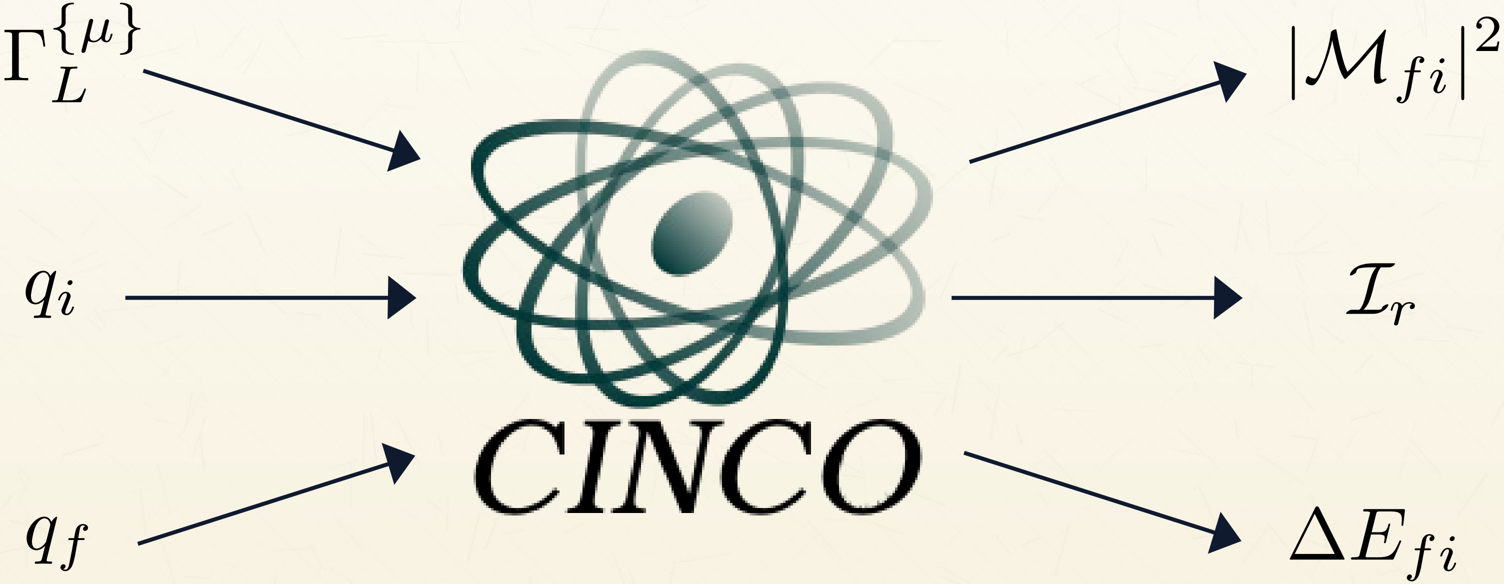

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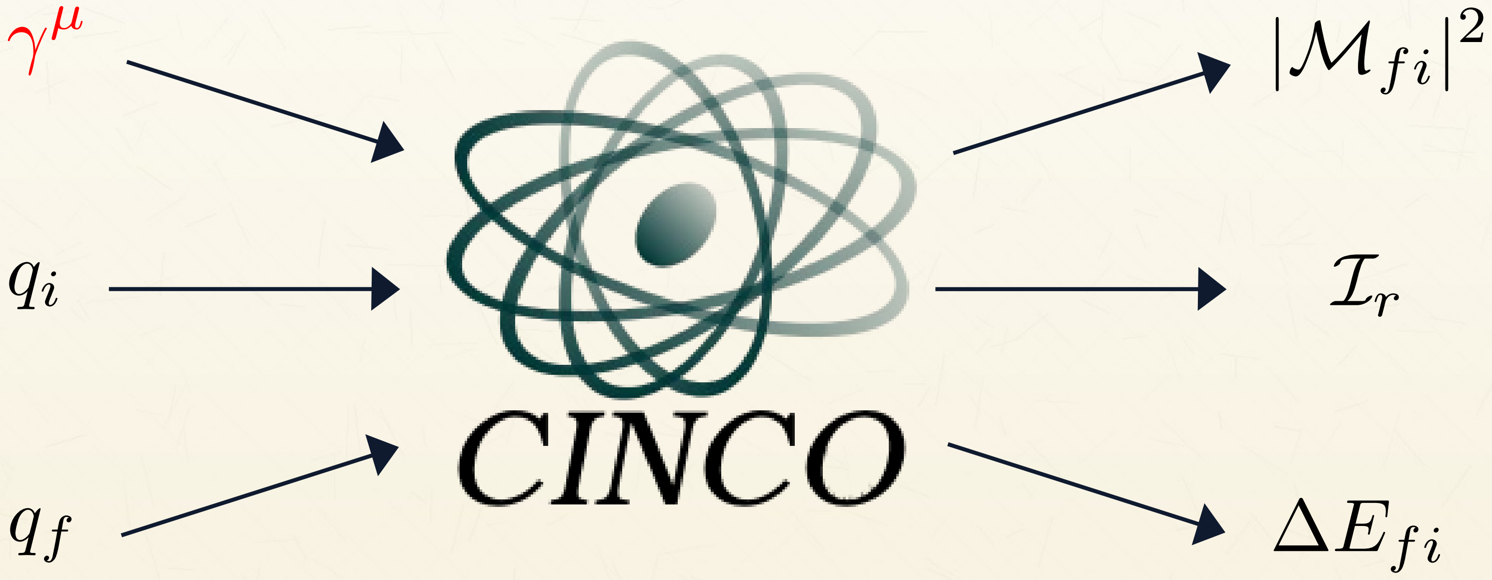

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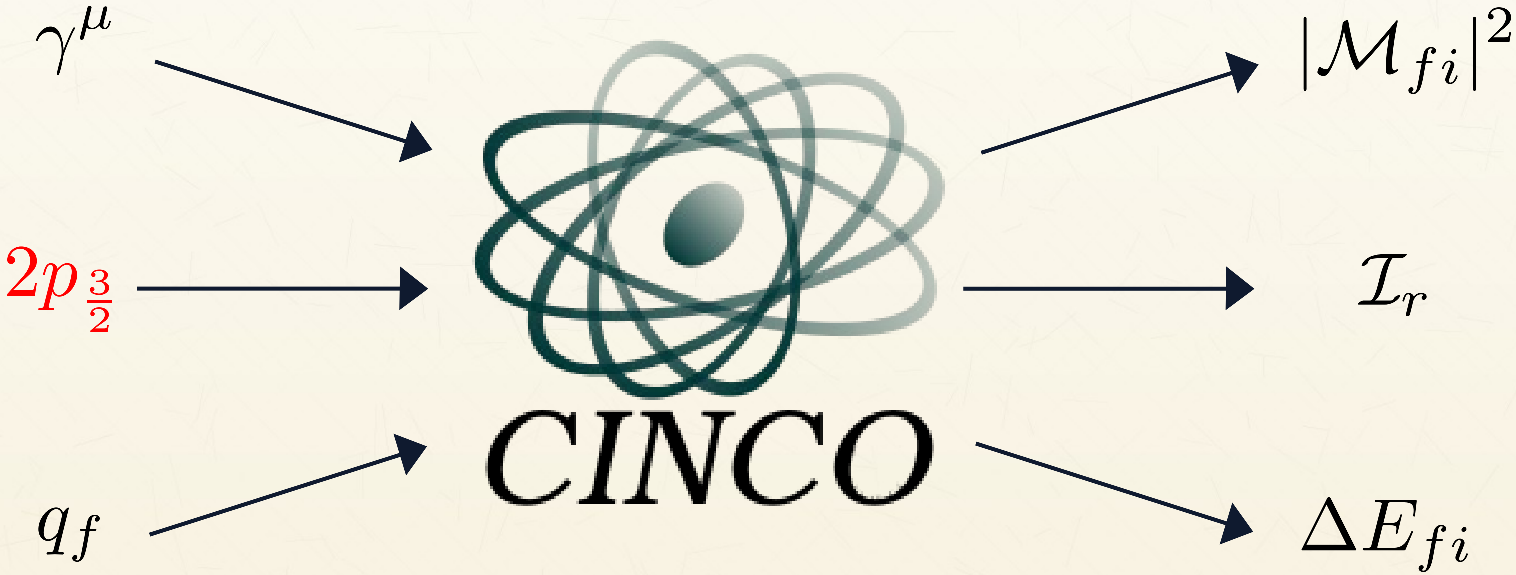

γ^μ

$2p_{\frac{3}{2}}$

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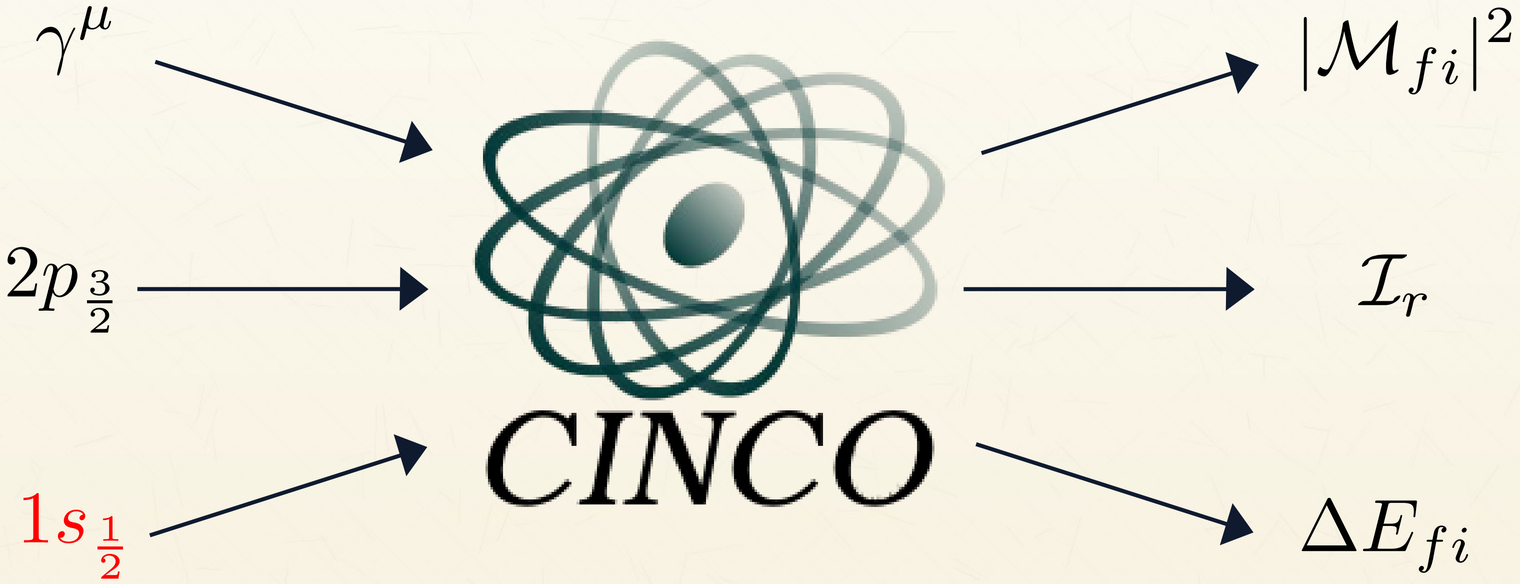

γ^μ

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$1s_{\frac{1}{2}}$

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Generalised atomic transitions

- Transition amplitude:

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Generalised atomic transitions

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$$\langle |\mathcal{M}_{fi}|^2 \rangle = \frac{4}{9} \left(\langle \vec{\mathcal{O}}_V \rangle^* \cdot \langle \vec{\mathcal{O}}_V \rangle \right) \mathcal{I}_{gf}^2$$

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Generalised atomic transitions

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$$\langle|\mathcal{M}_{fi}|^2\rangle = \frac{4}{9}\Big(\langle\vec{\mathcal{O}}_V\rangle^* \cdot \langle\vec{\mathcal{O}}_V\rangle\Big)\mathcal{I}_{gf}^2$$

- The expectation value (not done by CINCO):

$$\langle\mathcal{O}_{V,\mu}\rangle = \langle\gamma|A_\mu|0\rangle = \frac{e}{\sqrt{2E_\gamma}}\epsilon_\mu^*$$

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Generalised atomic transitions

- Transition amplitude:

$$\langle|\mathcal{M}_{fi}|^2\rangle = \frac{8\pi\alpha_{\mathrm{EM}}}{9E_\gamma}\,(\vec{\epsilon}\cdot\vec{\epsilon}^{\,*})\,\mathcal{I}_{gf}^2$$

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Generalised atomic transitions

- Phase space integral:

$$\Gamma_{2p_{3/2}\to 1s_{1/2}} = \frac{16\pi\alpha_{\mathrm{EM}}}{9}\Delta E_{fi}\mathcal{I}_{gf}^2$$

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Generalised atomic transitions

- Phase space integral:

$$\Gamma_{2p_{3/2}\to 1s_{1/2}} = 6.2650 \cdot 10^8\,\mathrm{s}^{-1}$$

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- Agrees to within 0.005%!

https://gitlab.com/JShergold/cinco

Contents

- Motivation: CνB and DM
- Generalised atomic transitions
- Pair absorption
- The future: molecules and ML

Pair absorption

- We're interested in very low energy particles:

Pair absorption

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Neutrinos

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- When does this win?

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Effective dim-6 coupling

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Pair absorption

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$$\textcolor{red}{n_\varphi} > E_{\text{thresh}}^3$$

$$\frac{\rho_{\text{DM}}}{m_\varphi}$$

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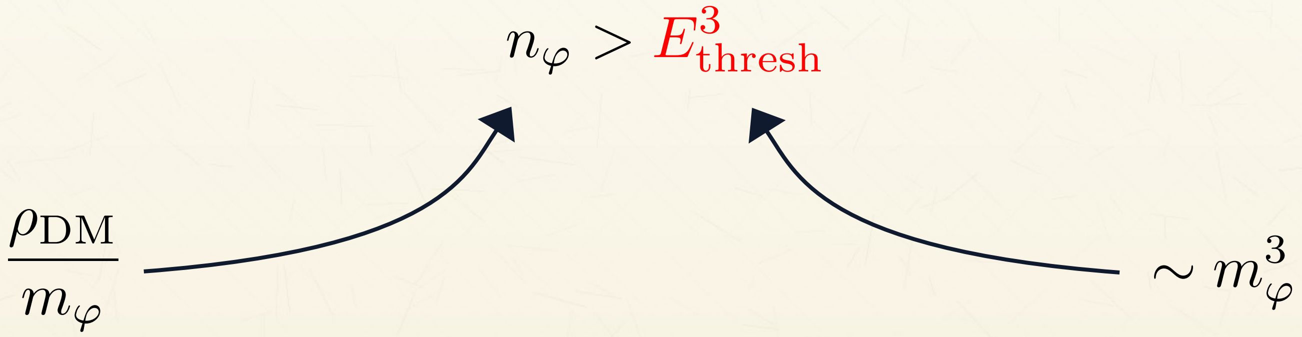

$$n_\varphi > E_{\text{thresh}}^3$$

$$\frac{\rho_{\text{DM}}}{m_\varphi}$$

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$$m_\varphi < \sqrt[4]{\rho_{\mathrm{DM}}}$$

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$0.3 - 0.4\,\mathrm{GeV\,cm}^{-3}$

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- Precisely where atomic transitions sit!

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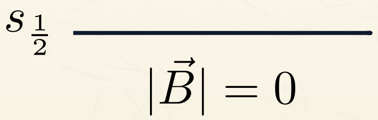

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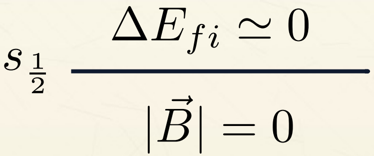

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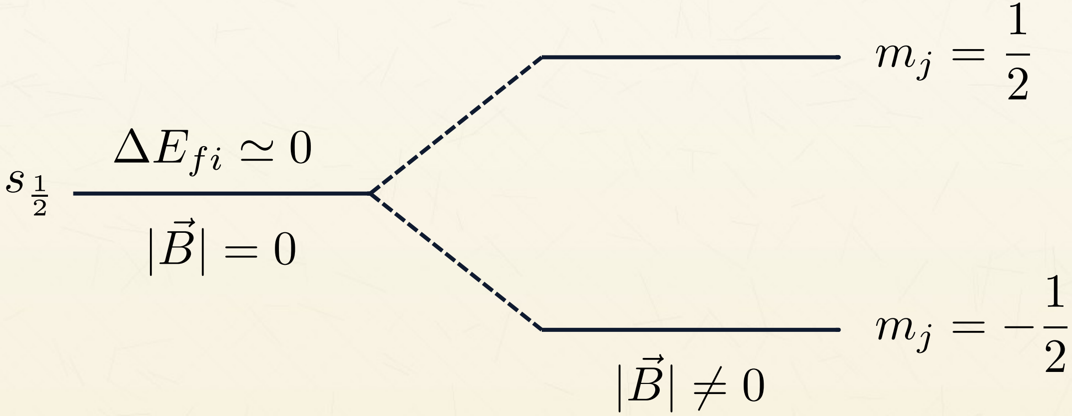

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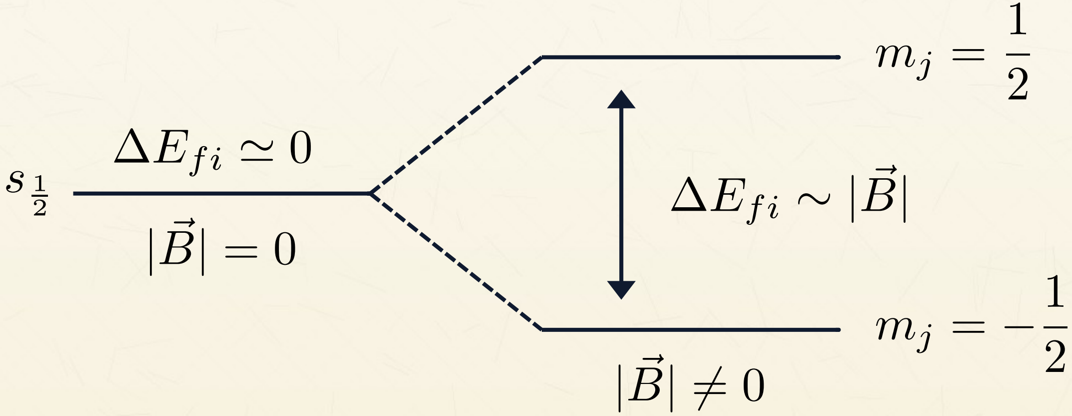

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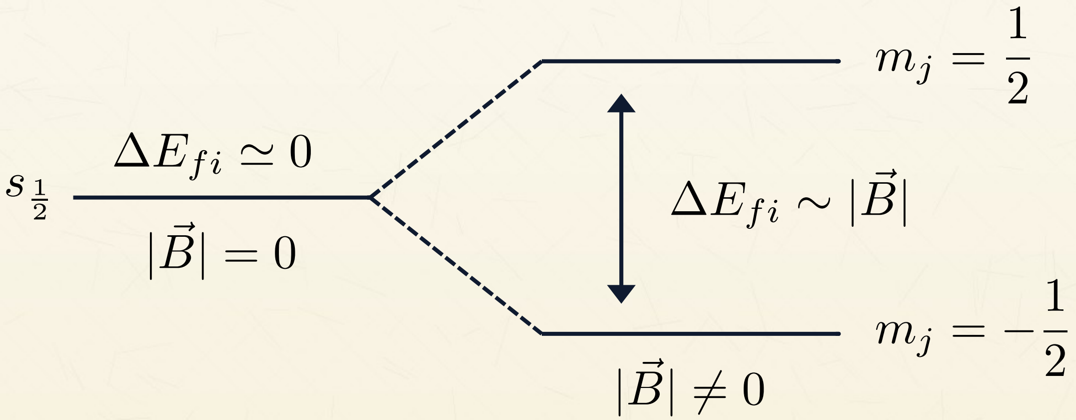

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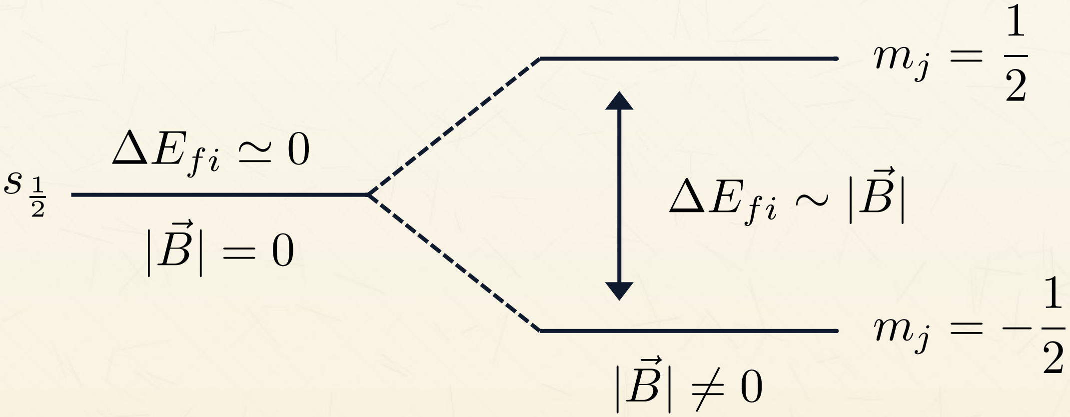

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$$\Delta n \neq 0$$

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Spin-flip	Fine structure	Principal
$\Delta m_j \neq 0$	$\Delta j \neq 0$	$\Delta n \neq 0$
$\Delta E_{fi} \sim \mathcal{O}(\mu\mathrm{eV} - \mathrm{meV})$	$\Delta E_{fi} \sim \mathcal{O}(\mathrm{meV} - \mathrm{eV})$	$\Delta E_{fi} \sim \mathcal{O}(\mathrm{eV})$
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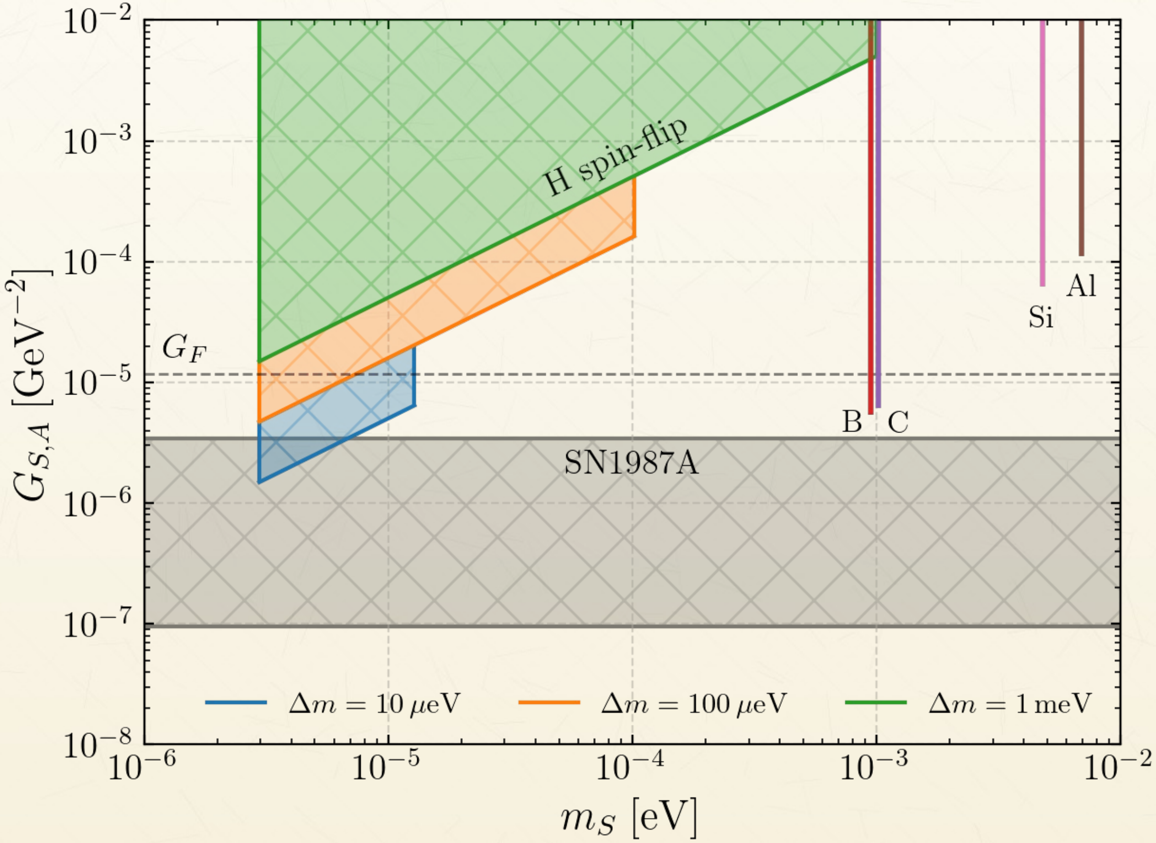
Pair absorption



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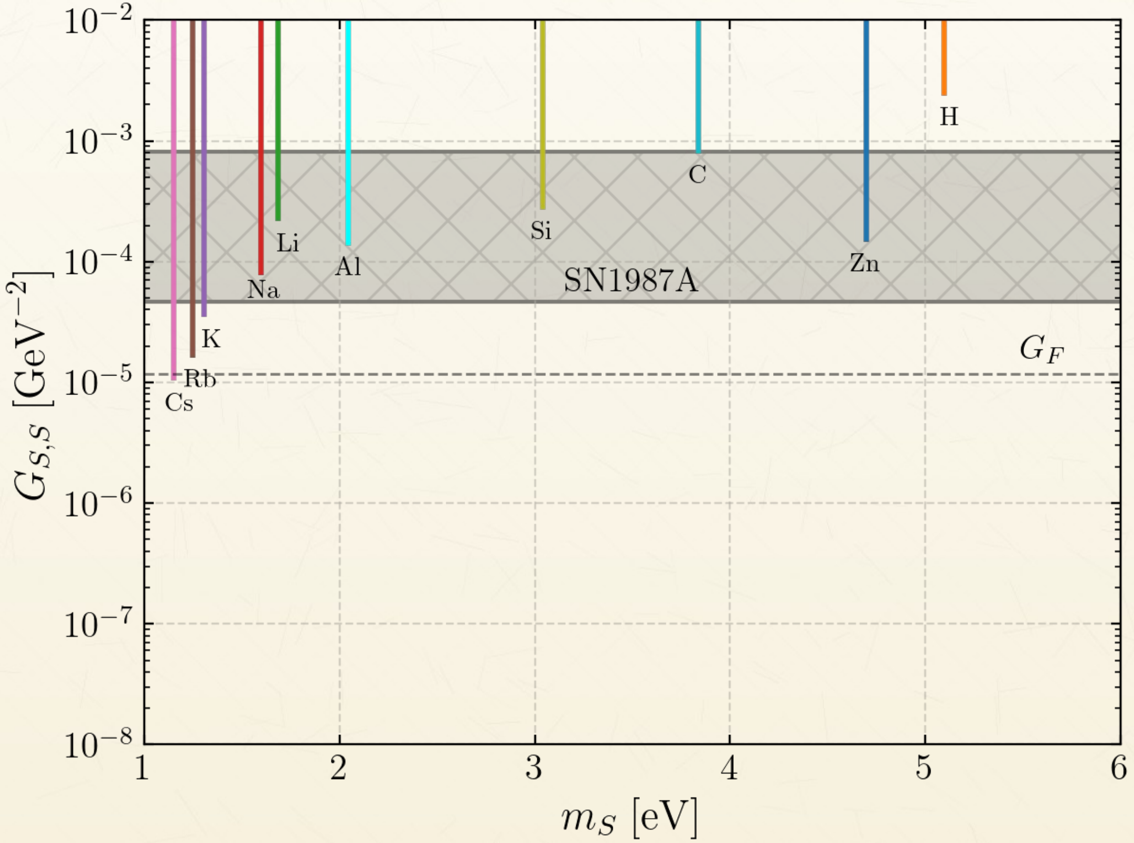

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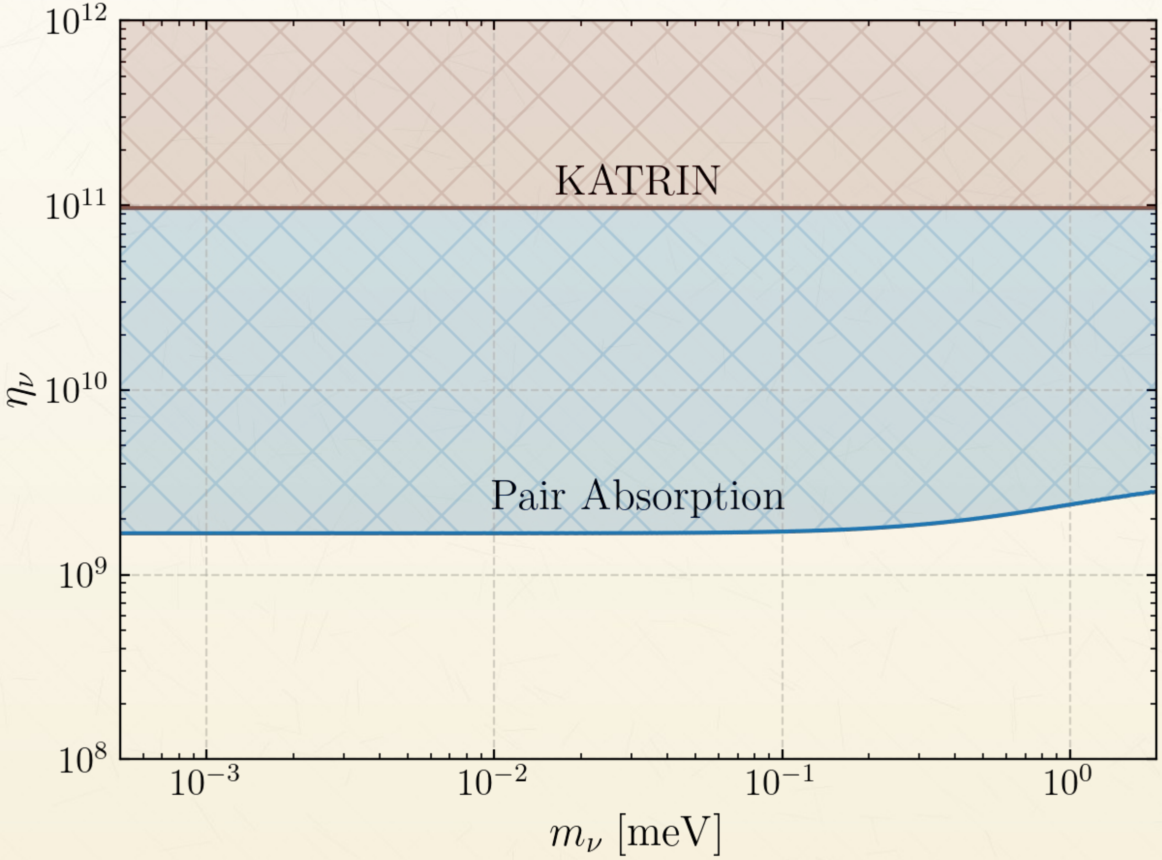

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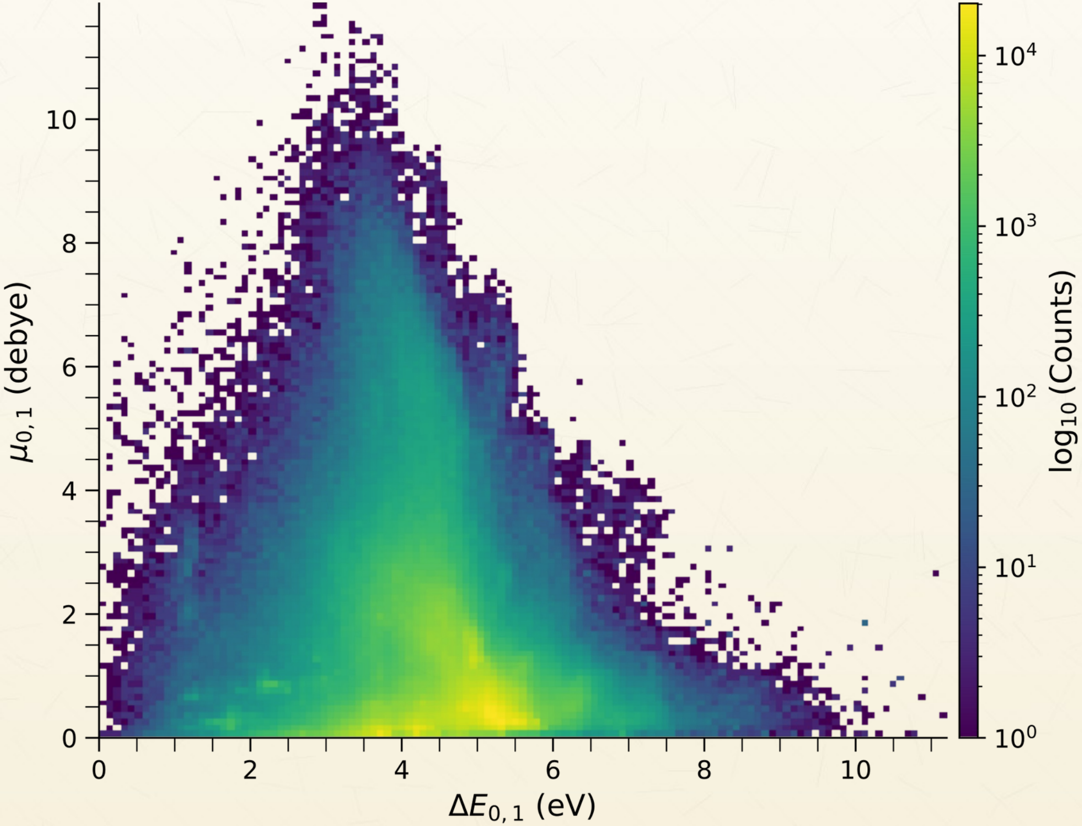
The future: molecules and ML



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- Watch this space!

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- Molecules and crystals, the future?

“I really don’t like the beige.” – My fiancée

Thank you

Any questions?